Proposed problems

PP. 16612. [17] If $x, y, z \in R$, then
$\left(\sum xy\right)^2 + 6\left(\sum xy\right)\left(\sum x\right) + \left(\sum x\right)^4 + 12 \geq 6xyz + 2\left(\sum xy\right)\left(\sum x\right)^2.$

Mihály Bencze

PP. 16613. Let n be a positive integer, and $k \in \{1, 2, ..., n\}$. Determine the number of distinct solutions of the equation $X_1 \cup X_2 \cup ... \cup X_k = \{1, 2, ..., n\}$.

Mihály Bencze

PP. 16614. Let ABC be a triangle
1). Prove that $\sum a \sin \frac{A}{2} \geq s$
2). Determine the minimum of the expression $\sum a \sin \frac{A}{4}$.

Mihály Bencze

PP. 16615. If $x, y, z > 0$, then $\sum \frac{(x+3y)y}{(x+y)^2} \geq 3$.

Mihály Bencze

PP. 16616. If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\prod\limits_{k=1}^{n} a_k = 1$, then
$\sum\limits_{k=1}^{n} \frac{n+a_k}{(1+a_k)^{n-1}} \geq \frac{n(n+1)}{2^{n-1}}$.

Mihály Bencze

PP. 16617. The tetrahedron $ABCD$ has circumsphere center O and tetrahedrons $OBCD$, $AOCD$, $ABOD$, $ABCO$ have circumspherecenters A_1, B_1, C_1, D_1. Compute $\frac{Vol[ABCD]}{[A_1B_1C_1D_1]}$.

Mihály Bencze

PP. 16618. Let $ABCDEF$ be a convex hexagon such that
$AD = x_1BC + y_1EF$, $BE = x_2AF + y_2CD$, $CF = x_3DE + y_3AB$. Determine all $x_k, y_k \in R$ $(k = 1, 2, 3)$ such that $\frac{AB}{DE} = \frac{CD}{AF} = \frac{EF}{BC}$.

Mihály Bencze

[17] *Solution should be mailed to editor until* 30.12.2013. *No problem is ever permanently closed. The editor is always pleased to consider for publication new solutions or new in sights on past problems.*

PP. 16619. We call a figure chrystal if it is made up of unit cubes connected by common edges. Prove that if there is an odd number of possible different chrystals consisting of n unit cubes on a $10 \times 10 \times 10$ board, then n is divisible by 8.

Mihály Bencze

PP. 16620. The quadrilateral $ABCD$ is inscribed in a circle. The tangents to the circle at A and C intersect at a point P not on BD and such that $PA^x = PB^y \cdot PD^z$. Determine all $x, y, z \in R$ such that BD passes trough the midpoint of AC.

Mihály Bencze

PP. 16621. Let ABC be a triangle such that $AC = \lambda BC$, and P is a point inside the triangle such that $PAB\measuredangle = \mu PBC\measuredangle$. If M is the midpoint of AB, then determine all $\lambda, \mu \in R$ such that $APM\measuredangle + BPC\measuredangle = 180^\circ$.

Mihály Bencze

PP. 16622. Solve in Z the equation
$\left(x^2 - 3x + 1\right)^2 + \left(y^2 + y - 1\right)^2 = \left(y^2 - 3x + 1\right)^2 + \left(x^2 + x - 1\right)^2.$

Mihály Bencze

PP. 16623. If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\sum\limits_{k=1}^{n} a_k = 1$, then for all $\alpha \in [0, 1]$ holds $\sum\limits_{cyclic} a_1 a_2^\alpha \le n^{-\alpha}$.

Mihály Bencze

PP. 16624. Let $a_k \in Z^*$ $(k = 1, 2, ..., n)$ such that $\sum\limits_{cyclic} \frac{a_1}{a_2} = n$. Determine all $n \in N^*$ for which $\prod\limits_{k=1}^{n} a_k$ is a perfect n power.

Mihály Bencze

PP. 16625. 1). In all acute triangle ABC holds
$\left(a^2 + h_a^2\right)\left(b^2 + h_b^2\right)\left(c^2 + h_c^2\right) \le (a + b)^2 (b + c)^2 (c + a)^2$

2). Determine all cevians c_a, c_b, c_c such that
$\left(a^2+c_a^2\right)\left(b^2+c_b^2\right)\left(c^2+c_c^2\right) \leq (a+b)^2(b+c)^2(c+a)^2$

Mihály Bencze

PP. 16626. If $a, b, c, d > 0$ then determine all $x, y \in R$ for which
$\min\left\{a^x-b^y, b^x-c^y, c^x-d^y, d^x-a^y\right\} \leq \frac{1}{4}$.

Mihály Bencze

PP. 16627. Compute $\int\limits_0^1 \frac{1-\left\{\frac{1}{x}\right\}^n}{1-\left\{\frac{1}{x}\right\}^k}dx$, where $n, k \in N^*$ and $\{\cdot\}$ denote the fractional part.

Mihály Bencze

PP. 16628. If $x, y, z > 0$ such that $xy+yz+zx=1$ and $a, b > 0$, then
1). $\sum \frac{1}{\sqrt{ax^2+byz}} \geq \frac{3\sqrt{3}}{\sqrt{a+b}}$
2). $\sum \frac{1}{(ax^2+byz)^\alpha} \geq 3\left(\frac{3}{a+b}\right)^\alpha$ for all $\alpha \in [0,1]$.

Mihály Bencze

PP. 16629. If $a_k > 0$ $(k=1,2,...,n)$, then $\sum\limits_{cyclic} \frac{a_1^{n-1}}{a_1^{n-1}+a_2a_3...a_n} \leq \frac{\sum\limits_{k=1}^n a_k}{2\sqrt[n]{\prod\limits_{k=1}^n a_k}}$.

Mihály Bencze

PP. 16630. If $x, y, z > 0$, then
$(x+y)(y+z)(z+x)(2x+y+z)(x+2y+z)(x+y+2z) \geq$
$16xyz(x+y+z)^3$.

Mihály Bencze

PP. 16631. If $x_k > 0$ $(k=1,2,...,n)$, then

$$\prod_{1\leq i<j\leq n}(x_i+x_j) \geq n\left(\prod_{k=1}^n x_k\right)\left(\sum_{k=1}^n x_k\right)^{\frac{n(n-3)}{2}}.$$

Mihály Bencze

PP. 16632. If $a_k, b_k > 0$ $(k=1,2,...,n)$ and $\lambda > 1$, then
1). $\left(\sum\limits_{k=1}^n (a_kb_k)^{\frac{2-\lambda}{\lambda}}\right)^\lambda \leq \left(\sum\limits_{k=1}^n a_kb_k^{1-\lambda}\right)\left(\sum\limits_{k=1}^n a_k^{-1}b_k^{\frac{1}{\lambda-1}}\right)^{\lambda-1}$

2). $\left(\sum\limits_{k=1}^{n} a_k b_k\right)^{\lambda} \leq \left(\sum\limits_{k=1}^{n} a_k b_k^{\lambda-1}\right)\left(\sum\limits_{k=1}^{n} a_k b_k^{\frac{1}{\lambda-1}}\right)^{\lambda-1}$

Mihály Bencze

PP. 16633. Determine all function $f : R \to R$ for which
$f\left((x-y)^3\right) = f^3(x) - 3f^2(x)\,y + 3f(x)\,y^2 - y^3$ for all $x, y \in R$.

Mihály Bencze

PP. 16634. Let a and b two natural numbers. Can both $a^3 + 4ab + 6b^2$ and $b^3 + 4ba + 6a^2$ be perfect squares?

Mihály Bencze

PP. 16635. If $a, b, c \in Z^*$ then
$\left(4a^2 + 3b^2 + 2c^2\right)\left|a\sqrt[3]{4} + b\sqrt[3]{2} + c\right| + \sum\left|(a+b)\left(a\sqrt{2} - b\right)\right| > \frac{5}{2}$.

Mihály Bencze

PP. 16636. Solve the equation $\sin^{2n+1} x + \cos^{2n+1} x = 2^{1-\frac{n}{2}}$, where $n \in N$.

Mihály Bencze

PP. 16637. Find all polynomial $P \in R[x]$ such that
$P^2(x) - P\left(y^2\right) = P\left(x^2\right) - P^2(y)$ for all $x, y \in R$.

Mihály Bencze

PP. 16638. Find all function $f : R \to R$ such that
$f(x^n - y^n) = xf^{n-1}(y) - yf^{n-1}(x)$ for all $x, y \in R$ and $n \in N$.

Mihály Bencze

PP. 16639. If $f_a(n) = n + \max\left\{k \in N : a^{a^k} \leq n \cdot a^n\right\}$ where $a \in N$, $a \geq 2$, then solve the equations:
1). $f_a(n) = f_b(m)$
2). $f_a(n) = d(na)$
3). $f_a(n) = \sigma(a+n)$
4). $f_a(n) = \Phi(a+n)$

Mihály Bencze

PP. 16640. Find all continuous function $f : R \to R$ such that
$f\left(\prod_{k=1}^{2n} x_k\right) = f\left(\frac{1}{2n}\sum_{k=1}^{2n} x_k^{2n}\right) + \sum_{1\le i<j\le 2n} (x_i - x_j)^2$ for all $x_i \in R$
$(i = 1, 2, ..., 2n)$.

Mihály Bencze

PP. 16641. Find all $f : R \to R$ such that
$f(x+y) f(x-y) + f(x+2y) f(x-2y) = f\left(x\sqrt{2} + y\sqrt{5}\right) f\left(x\sqrt{2} - y\sqrt{5}\right)$
for all $x, y \in R$.

Mihály Bencze

PP. 16642. Prove that for any $x > 0$, there are integers $a_k \in Z$
$(k = 1, 2, ..., n)$ such that $x > \left(\sum_{cyclic} a_1^{n-1} a_2\right)\left(\prod_{k=1}^{n} a_k\right)^{-1}$.

Mihály Bencze

PP. 16643. Find all $f : R \to R$ such that
$f(x+y)\left(f\left(x^2\right) - f(xy) + f^2(y)\right) = x^3 - f^3(y)$ for all $x, y \in R$.

Mihály Bencze

PP. 16644. Find all $f : R \to R$ such that
$(f(x) + f(y))\left(f^2(y) - f(y) f(z) + f^2(z)\right) =$
$xf\left(x^2 + z^2\right) - (y - z) f(yz) + y^3 - f(x) f(y) f(z)$ for all $x, y, z \in R$.

Mihály Bencze

PP. 16645. Let $a_k \in R$ $(k = 2, 3, ..., n)$ fixed, no all zeros. Find all functions
$f : R \to R$ such that $\sum_{k=2}^{n} a_k f(x_1 x_k) - f(x_1) f\left(\prod_{k=2}^{n} x_k\right) \ge \frac{(n-2)\left(\sum_{k=2}^{n} a_k\right)^2}{(n-1)^2}$ for
all $x_1, x_2, ..., x_n \in R$.

Mihály Bencze

PP. 16646. Find all continuous functions $f : R \to R$ such that
$f(x + f(y + f(z))) + f(y + f(z + f(x))) + f(z + f(x + f(y))) =$
$3(f(x) + f(y) + f(z))$ for all $x, y, z \in R$.

Mihály Bencze

PP. 16647. Compute

1). $\sum\limits_{i_1=1}^{n} \ldots \sum\limits_{i_k=1}^{n} \cos \frac{k\left(i_1^k+\ldots+i_k^k\right)}{n}$

2). $\sum\limits_{i_1=1}^{n} \ldots \sum\limits_{i_k=1}^{n} \sin \frac{k\left(i_1^k+\ldots+i_k^k\right)}{n}$

Mihály Bencze

PP. 16648. Determine all $a, b > 0$ such $\int\limits_0^{\pi} (a \sin x + b \cos x)^3 \, dx = \frac{4a^2 b\sqrt{6}}{3}$.

Mihály Bencze

PP. 16649. Determine all functions $f, g : N \to N$ such that

$$\Big(\underbrace{f \circ \ldots \circ f}_{g(n) \text{ time}}\Big)(m) = f(n) + g(m) \text{ and } \Big(\underbrace{g \circ \ldots \circ g}_{f(n) \text{ time}}\Big)(m) = g(n) + f(m).$$

Mihály Bencze

PP. 16650. Determine all functions $f : R \to R$ such that
$f(x^n f(y) + y^n f(x)) = x f^n(y) + y f^n(x)$ for all $x, y \in R$, where $n \in N$.

Mihály Bencze

PP. 16651. The triangle ABC with sides a, b, c is equilateral if and only if
$\frac{xa+(1-x)b}{c} = \frac{xb+(1-x)c}{a} = \frac{xc+(1-x)a}{b}$ for all $x \in R$.

Mihály Bencze

PP. 16652. Solve in N the equation $\frac{x}{y} = \frac{\left(x^2-y\right)^z+\left(y^2-x\right)^z}{(xz-y)^2+(yz-x)^2}$.

Mihály Bencze

PP. 16653. Determine the best constant $a \in \left[\frac{1}{6}, \frac{1}{3}\right)$ such that
$x - \sin x \leq ax^3 \leq tgx - x$ for all $x \in \left[0, \frac{\pi}{2}\right]$.

Mihály Bencze

PP. 16654. If $x \in \left[0, \frac{\pi}{2}\right]$, then $\frac{3}{2} + \frac{\pi^2}{9} + (x - \sin x)(tgx - x) \leq \frac{\pi\sqrt{3}}{2}$.

Mihály Bencze

PP. 16655. If $a, b, c > 0$, then $\left(\sum a^2\right)^2 \geq \left(\sum ab\right)^2 + \frac{1}{3}\left(\sum \left|a^2 - bc\right|\right)^2$.

Mihály Bencze

PP. 16656. Solve the following system: $\begin{cases} \log_a\left(x + b^{\log_c y}\right) = \log_c z \\ \log_a\left(y + b^{\log_c z}\right) = \log_c x \\ \log_a\left(z + b^{\log_a x}\right) = \log_c y \end{cases}$,
where $a, b, c > 1$.

Mihály Bencze

PP. 16657. Solve in N the equation $x^3 + y^5 + z^{15} + t^{75} = u^{98}$.

Mihály Bencze

PP. 16658. Determine all continuous functions $f : R \to R$ such that $f(nx) = \frac{nf(x)}{1+f^n(x)}$ for all $x \in R$, where $n \in N^*$ is given.

Mihály Bencze

PP. 16659. For each positive integer n define $f(n)$ as the number of digits 0 in its decimal representation. For example $f(1) = 0$, $f(102) = 1$, $f(2009) = 2$.

1). Compute $S(n) = \sum\limits_{k=1}^{n} a^{f(k)}$, where $a \geq 2$, $a \in N$.

2). Compute $\sum\limits_{n=1}^{\infty} \frac{1}{1+S^{\alpha}(n)}$, where $\alpha \in N$.

Mihály Bencze

PP. 16660. Determine all continuous functions $f : R^* \to R$ for which $f\left(x + \frac{1}{y^2}\right) + f\left(y + \frac{1}{x^2}\right) = f\left(x + \frac{1}{x^2}\right) + f\left(y + \frac{1}{y^2}\right)$ for all $x, y \in R^*$.

Mihály Bencze

PP. 16661. Prove that $\prod\limits_{i=k}^{n} \binom{i}{k}^i \leq \left(\frac{2(n+1)\binom{n+1}{k+1} - 2\binom{n+2}{k+2}}{(n+k)(n-k+1)}\right)^{\frac{(n+k)(n-k+1)}{2}}$.

Mihály Bencze

PP. 16662. If $x_k > 0$ $(k = 1, 2, ..., n)$, then
$3\sum\limits_{k=1}^{n} x_k^3 + 6\sum\limits_{cyclic} x_1x_2x_3 \geq \frac{1}{4}\sum\limits_{cyclic}(x_1 + x_2 + x_3)^3.$

Mihály Bencze

PP. 16663. Prove that $\left(\sum\limits_{k=1}^{n}\sqrt{\binom{n}{k}}\right)^{2n}\left(\prod\limits_{k=0}^{n+1}\binom{n+1}{k}\right) \leq 2^n\left(2^n - 1\right)^{2n}.$

Mihály Bencze

PP. 16664. If $a_k \in (0, 1]$ $(k = 1, 2, ..., n)$, then $\frac{1}{\sum\limits_{k=1}^{n} a_k} \geq \frac{1}{n} + \prod\limits_{k=1}^{n}(1 - a_k).$

Mihály Bencze

PP. 16665. Let $A \in \mathcal{M}_{2\times 2}(R)$ be the matrix defined by
$A = \frac{\pi}{4}\begin{pmatrix} 7 & -3 \\ -3 & 7 \end{pmatrix}.$
Express $\sin(A)$ in a close form.

José Luis Díaz-Barrero

PP. 16666. Let x_1, x_2, x_3 be the roots of the equation $x^3 - 2x^2 - x + 1 = 0$. Find the equation which roots are $y_1 = x_1 + \dfrac{1}{x_1}$, $y_2 = x_2 + \dfrac{1}{x_2}$ and
$y_3 = x_3 + \dfrac{1}{x_3}$.

José Luis Díaz-Barrero

PP. 16667. Let $x_k > 0$, $a_k > 0$, $1 \leq k \leq n$, be real numbers with
$a_1 + a_2 + \ldots + a_n = 1$. Show that $\frac{1}{n}\left(\sum\limits_{k=1}^{n} x_k\right)^3 \leq \sum\limits_{k=1}^{n}\frac{x_k^3}{a_k}$.

José Luis Díaz-Barrero

PP. 16668. Let a, b, c be real numbers such that $0 < a, b, c \leq 1$. Prove that

$\sqrt[3]{\left(\frac{1}{a} + b\right)^3 + \left(\frac{1}{b} + c\right)^3 + \left(\frac{1}{c} + a\right)^3} \geq 2\sqrt[3]{3}.$

José Luis Díaz-Barrero

PP. 16669. Let $a_1, a_2, \ldots, a_n$ be nonnegative real numbers. Show that $\frac{1}{n}\left(\sum\limits_{k=1}^{n} \ln(1+a_k)\right)^2 \leq \sum\limits_{k=1}^{n} a_k^2$.

José Luis Díaz-Barrero

PP. 16670. Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be real numbers such that $1 + \sum\limits_{k=1}^{n} \cos^2 \alpha_k = n$. Prove that $\sum\limits_{1\leq i<j\leq n} \tan\alpha_i \tan\alpha_j \leq \frac{n}{2}$.

José Luis Díaz-Barrero

PP. 16671. Compute $\lim\limits_{n\to\infty} \ln\left[\frac{1}{2^n}\prod_{k=1}^{n}\left(2+\frac{k}{n^2}\right)\right]$.

José Luis Díaz-Barrero

PP. 16672. Find the loci of the orthocenters of the triangles inscribed in an equilateral hyperbola.

José Luis Díaz-Barrero

PP. 16673. Let x, y, z be three distinct positive real numbers. Prove that $\frac{1}{\max\{x,y,z\}} < \sum\limits_{cyclic} \frac{\ln x^{2x}}{(x-y)(x-z)} < \frac{1}{\min\{x,y,z\}}$.

José Luis Díaz-Barrero

PP. 16674. If $x \in \left(0, \frac{\pi}{2}\right)$, then $0 < \frac{\sin x}{2+(n\cos x)^{-n\cos x}} + \frac{\cos x}{2+(n\sin x)^{-n\sin x}} < 1$ for all $n \in N$.

Mihály Bencze

PP. 16675. Let M be a point in the interior of tetrahedron $ABCD$, and let r be the inradius of insphere. Prove that $\max\{MA, MB, MC, MD\} \geq 3r$.

Mihály Bencze

PP. 16676. Let $ABCD$ be a convex quadrilateral such that AC and BD intersect in right angles at P, and $M \in AB$, $N \in BC$, $K \in CD$, $L \in DA$ such that $AM = xMB$, $BN = yNC$, $CK = zKD$, $LD = tDA$.

Determine all $x, y, z, t > 0$ such that the circles
$(PMN), (PNK), (PKL), (PLM)$ are congruent if and only if $ABCD$ is cyclic.

Mihály Bencze

PP. 16677. If $x, y, z > 0$, then $\sum \sqrt{9(x^2+y^2+z^2) - 14xy + 18(x+y)z} +$
$+ \sum \sqrt{x^2+y^2+z^2 - xy + 2(x+y)z} \geq \left(z+\sqrt{6}\right)(x+y+z)$.

Mihály Bencze

PP. 16678. Determine all $\alpha > 0$ such that
$\left(\frac{1}{\alpha} + \sum \frac{x}{y+z}\right)\left(\sum x\right) \leq \alpha \sum \frac{x^\alpha + yz}{y+z} \leq \left(\frac{1}{\alpha} + \sum \frac{x^\alpha}{y^\alpha + z^\alpha}\right)\left(\sum x\right)$ for all $x, y, z > 0$.

Mihály Bencze

PP. 16679. Determine all function $f : R \to (0, +\infty)$ such that
$3f(x+y+z) - f(-x+y+z) - f(x-y+z) - f(x+y-z) =$
$= 8\left(\sqrt{f(x)f(y)} + \sqrt{f(y)f(z)} + \sqrt{f(z)f(x)}\right)$ for all $x, y, z \in R$.

Mihály Bencze

PP. 16680. In all triangle ABC holds
$\sum \frac{\sin A}{4\sin^2 A + \cos A + \cos(B-C)} \leq \sqrt{\frac{R(s^2+r^2+4Rr)}{8s^2 r}}$.

Mihály Bencze

PP. 16681. 1). If $x \in R$, then $\left(1+\sin^6 x\right)\left(1+\cos^6 x\right) \leq 4$
2). If $x, y, z \in R$ and $\cos^2 x + \cos^2 y + \cos^2 z = 2$,then
$tgxtgy + tgytgz + tgztgx \leq \frac{3}{2}$.

Mihály Bencze

PP. 16682. Determine all $n \in N^*$ such that $n^2\left(n^2+16\right)$ is divisible by 720 and $n-1, n+1$ are both prime.

Mihály Bencze

PP. 16683. Let ABC be a triangle with $AC > AB$. The point M lies on the side BA extended A, and the point N lies on the side CA in such a way that $BM = \alpha CA$ and $CN = \beta BA$.

The line MN meets the perpendicular bisector of side BC at P. Determine all $x, y > 0$ such that $BPC\measuredangle + BAC\measuredangle = \pi$.

Mihály Bencze

PP. 16684. Consider the set $M = \{1, 2, 3, ..., k^n\}$, $(k, n \geq 2)$. Find k and the number of subsets P of M, such that if the sum of k elements of M is a power of k, then exactly one of them belongs to P.

Mihály Bencze

PP. 16685. If $x, y, z > 0$ such that $\frac{1}{chx} + \frac{1}{chy} + \frac{1}{chz} = 2$, then $shxshy + shyshz + shzshx \leq \frac{3}{2}$.

Mihály Bencze

PP. 16686. Let ABC be a triangle, R the circumradius, r_a the excircle radius etc. An excircle of the given triangle ABC touch the side BC, AC, AB in M, N, L, respectively. Show that the circumcentre of triangle ABC is the orthocenters of triangle MNL if and only if $R = r_a$.

Mihály Bencze

PP. 16687. If $a_k > 0$ $(k = 1, 2, ..., n)$,then $\sum\limits_{cyclic} \left(\frac{a_1 a_3}{a_2^2}\right)^{n-2} \geq \sum\limits_{cyclic} \frac{a_1}{a_2}$.

Mihály Bencze

PP. 16688. If $x, a, b \in (0, 1)$, then
$\prod\limits_{k=1}^{n} \left(1 - ax^k\right) + \prod\limits_{k=1}^{n} \left(1 - bx^k\right) \geq 2 - \frac{(a+b)x}{1-x}$.

Mihály Bencze

PP. 16689. If $x, y \geq 0$, then

1). $\frac{1}{2} shx + \frac{\left(1-ch^2x\right)^2}{shx+\sqrt{ch^2x-1}} \leq \frac{ch^4x}{chx+\sqrt{2+ch^2x}}$
2). $\frac{1}{chx} + \frac{1}{chy} \geq \frac{2}{\sqrt{1+(shxshy)^2}}$

Mihály Bencze

PP. 16690. If $x \in R\backslash\{-1,1\}$, then
$$2\left|\frac{x}{1-x^2}\right| + \frac{1}{2}\left|\frac{1-x^2}{x}\right| + \sqrt{1+\frac{4x(1-x^2)}{(1+x^2)^2}} \geq 2+\sqrt{2}.$$

Mihály Bencze

PP. 16691. Let ABC be a triangle which have at most one angle exceeding πx. Determine all $x \in (0,1)$ such that $Area\,[ABC] \leq \frac{\sqrt{3}\left(s^2+r^2+4Rr\right)^2}{24(s^2-r^2-4Rr)}$.

Mihály Bencze

PP. 16692. If $a_k > 0$ $(k = 1, 2, ..., n)$, then
$$\sum_{cyclic}\frac{a_1}{a_2} + \sqrt{1+\frac{n\prod\limits_{k=1}^{n} a_k}{\sum\limits_{k=1}^{n} a_k^n}} \geq n+\sqrt{2}.$$

Mihály Bencze

PP. 16693. Solve the system $\begin{cases} tgx + ctgy + \sqrt{1+\sin 2z} \geq 2+\sqrt{2} \\ tgy + ctgz + \sqrt{1+\sin 2x} \geq 2+\sqrt{2} \\ tgz + ctgx + \sqrt{1+\sin 2y} \geq 2+\sqrt{2} \end{cases}$.

Mihály Bencze

PP. 16694. If $x > 0$, then $\frac{ch^2x}{shx} + \frac{1+shx}{chx} \geq 2+\sqrt{2}$.

Mihály Bencze

PP. 16695. Let $ABCD$ be a parallelogram. A line through D meets the segment AC in G, the side BC in F, and the line AB in E. Determine all $\alpha > 0$ such that $Area\,[BEF] = \alpha Area\,[CGF]$ and $\frac{AG}{GC} = \frac{1+\sqrt{5}}{2}$.

Mihály Bencze

PP. 16696. Let B and C two point on a circle, and TU a chord perpendicular to BC. The points P and M are chosen on TU and CU such that $CU \| PB$ and $CT \| MP$. Let k be the intersection of the lines CT and PB. If B is fixed, then determine all points C on the circle such that $PB \perp MK$.

Mihály Bencze

PP. 16697. A triangle is said to be multiplicative if the product of the lengths of two of its sides equals the length of the third side. Let $A_1A_2...A_n$ be a regular polygon inscribed in a sphere. The $n-3$ diagonals from the vertex A_1 divide the triangle $A_nA_1A_2$ into $n-2$ smaller triangles $AA_nB_1, AB_1B_2, ..., AB_{n-2}B_2$. Determine all points M on the sphere for which $MA_nB_1, MB_1B_2, ..., MB_{n-2}B_2$ are multiplicative triangles.

Mihály Bencze

PP. 16698. The tetrahedron $ABCD$ has B, C, D fixed vertices. Let H and G be the orthocentre and the centroid of tetrahedron $ABCD$. Find the locus of A such that the midpoint K of the segment HG lies on the plane BCD.

Mihály Bencze

PP. 16699. If $a_k \in R$ $(k = 1, 2, ..., n)$ and $a > 0$, $F(x) = \sum\limits_{k=1}^{n} a^{k-1} f(a_k + x)$, then determine all functions $f : R \to R$ such that from $F(x_1) = F(x_2)$ holds $x_1 - x_2 = m\pi$ $(m \in Z)$.

Mihály Bencze

PP. 16700. Determine all $x_k \in Z$ $(k = 1, 2, ..., n)$ and $m \in N$ such that $\frac{x_1^m - 1}{x_2 + 1} + \frac{x_2^m - 1}{x_3 + 1} + ... + \frac{x_n^m - 1}{x_1 + 1}$ is integer.

Mihály Bencze

PP. 16701. Find all polynomials $P(x)$ of degree $2k+1$ such that for all $x_i > 0$ $(i = 1, 2, ..., n)$ holds $P\left(\sum\limits_{i=1}^{n} x_i\right) \geq \sum\limits_{i=1}^{n} P(x_i)$.

Mihály Bencze

PP. 16702. Determine all $a, b > 0$ such that $\{an\} \geq \frac{b}{n}$ for all $n \in N^*$, where $\{\cdot\}$ denote the fractional part.

Mihály Bencze

PP. 16703. Determine all $a, b > 0$ such that $\{an\} + \{bn\} \geq \frac{a+b}{n}$ for all $n \in N^*$, where $\{\cdot\}$ denote the fractional part.

Mihály Bencze

PP. 16704. Let A, B, C, D be four point on a sphere (S). Prove that exist for points M_1, M_2, M_3, M_4 on (S) which form a tetrahedron such that $\frac{AM_k}{BM_k} = \frac{DM_k}{CM_k}$ $(k = 1, 2, 3, 4)$.

Mihály Bencze

PP. 16705. Triangle ABC is isosceles with $AB = AC$. The line (d) passes through A and is parallel to BC. The points P and Q are on the perpendicular bisectors of AB and AC, respectively, and such that $PQ \perp BC$. Determine all points M and N on (d) such that $APM\measuredangle + AQN\measuredangle = \pi$ and $\frac{1}{AM} + \frac{1}{AN} \leq \frac{2}{AB}$.

Mihály Bencze

PP. 16706. If $a_k, b_k > 0$ $(k = 1, 2, ..., n)$, then

$$\sum_{k=1}^{n} \frac{a_k^3 b_k^3}{(a_k+a_{k+1})(b_k+b_{k+1})} \geq \frac{\left(\sum\limits_{k=1}^{n} a_k b_k\right)^3}{4\left(\sum\limits_{k=1}^{n} a_k\right)\left(\sum\limits_{k=1}^{n} b_k\right)}.$$

Mihály Bencze

PP. 16707. Let $S_1, S_2, ..., S_n$ be spheres in R^3 such that among any k of then there are two with nonempty intersection. Prove that one can choose $k - 1$ points in R^3 such that each of the spheres contains at least one of the chosen points.

Mihály Bencze

PP. 16708. Determine all $n \in N^*$ such that $4\left(1^3 + 2^3 + ... + n^3\right) =$

$$\left(1^4 + 2^4 + ... + n^4\right) + 1^3 + \left(1^3 + 2^3\right) + \left(1^3 + 2^3 + 3^3\right) + ... + \left(1^3 + 2^3 + ... + n^3\right).$$

Mihály Bencze

PP. 16709. If ζ denote the Riemann function then for all $x \in (1, 2]$ we have:

1). $x \geq (x-1)\zeta(x) \geq 2^{1-x} + x - 1$
2). $\frac{1}{2} \leq (x-1)\zeta(x) \leq 2$

Mihály Bencze

PP. 16710. Given integers $m, n > 2$ then determine all $p \in N^*$ for which there is a sequence of integers $a_0, a_1, ..., a_k$ such that $a_0 = m$, $a_k = n$ and $(a_i + a_{i+1} + ... + a_{i+p}) \mid (a_i a_{i+1} ... a_{i+p} + 1)$ for each $i \in \{0, 1, ..., k-1\}$.

Mihály Bencze

PP. 16711. Solve the following system:

$$\begin{cases} 2\sqrt{2} + x = 3\sqrt{3} + \left(\sqrt{2} - \sqrt{3}\right)\frac{y^2+1}{y^2-1} \\ 2\sqrt{2} + y = 3\sqrt{3} + \left(\sqrt{2} - \sqrt{3}\right)\frac{z^2+1}{z^2-1} \\ 2\sqrt{2} + z = 3\sqrt{3} + \left(\sqrt{2} - \sqrt{3}\right)\frac{x^2+1}{x^2-1} \end{cases}.$$

Mihály Bencze

PP. 16712. The intersection of the ellipsoid $x^2 + y^2 + \left(\frac{z}{a}\right)^2 = 1$ and the plane $x + y + bz = 0$ is an ellipse. Determine all $a, b \in Q$ for which $\frac{1}{\pi}$ of ellipse area is rational.

Mihály Bencze

PP. 16713. If $a, b, c, d > 0$ then
$\frac{(a+c)^2+(a+d)^2}{a+b} + \frac{(b+c)^2+(a+d)^2}{a+c} + \frac{(b+d)^2+(c+d)^2}{a+d} + \frac{(c+d)^2}{b+d} + \frac{(b+d)^2}{b+c} \geq$
$\geq 4\left(c + 3d - 3a - b\right)$.

Mihály Bencze

PP. 16714. If $a, b, c > 0$ and $\sum \frac{a}{b} = 1$, then $\sum \frac{lg\left(1 - 2\sum \frac{a}{c}\right)}{lga} \leq 0$.

Mihály Bencze

PP. 16715. If $x, y, z > 0$, then $\sqrt{xyz}\sum \sqrt{x} + 2\left(\sum x\right)\left(\sum xy\right) \leq \left(\sum x\right)^2$.

Mihály Bencze

PP. 16716. If $x_k > 0$ $(k = 1, 2, ..., n)$, then $\sum\limits_{k=1}^{n} \frac{1}{1+\sqrt{1+\frac{S}{x_k}}} \leq \frac{n}{1+\sqrt{n+1}}$, where $S = \sum\limits_{i=1}^{n} x_i$.

Mihály Bencze

PP. 16717. If $a_i > 0$ $(i = 1, 2, ..., n)$, then determine all $k \in N^*$ for which
$\frac{a_1+...+a_k}{a_k+...+a_{2k-1}} + \sqrt{\frac{a_k+...+a_{2k-1}}{a_{k+1}+...+a_{2k}}} + ... + \sqrt[n]{\frac{a_n+...+a_{k-1}}{a_{k-1}+...+a_{2k-2}}} \leq n.$

Mihály Bencze

PP. 16718. If $a_0, a_n > 0$ and $a_n^2 < a_n - a_{n+1}$ for all $n \geq 1$, then compute
$\lim\limits_{n\to\infty} \frac{1}{n}\left(\sum\limits_{k=1}^{n} ka_k\right).$

Mihály Bencze

PP. 16719. If $a_1 \in \left(0, \frac{1}{5}\right)$ and $a_{n+1} + 5a_n^2 = a_n$ for all $n \geq 1$, then compute
$\lim\limits_{n\to\infty} n\left(\frac{1}{5} - na_n\right).$

Mihály Bencze

PP. 16720. If $a, b, c > 0$, then determine all $k \in N$ for which
$\sum \frac{ab^{\frac{1}{k}}}{a^{\frac{1}{k}}\left(ab+c^k\right)} \leq \frac{k+1}{k}.$

Mihály Bencze

PP. 16721. Solve the following system:
$$\begin{cases} \left(x^2 - 2x + 10\right)\left(-y^2 + 2y + 2\right) \leq 27 \\ \left(y^2 - 2y + 10\right)\left(-z^2 + 2z + 2\right) \leq 27 \\ \left(z^2 - 2z + 10\right)\left(-x^2 + 2x + 2\right) \leq 27 \end{cases}.$$

Mihály Bencze

PP. 16722. Determine all triangle ABC such that $\sum \frac{a}{\sqrt{s-a}} \geq \sqrt{6}$.

Mihály Bencze

PP. 16723. Solve the following system: $\begin{cases} 2x^y = y^z + z^x \\ 2y^z = z^x + x^y \\ 2z^x = x^y + y^z \end{cases}.$

Mihály Bencze

PP. 16724. Let $d_1, d_2, ..., d_k$ be the natural divisors of number m, and
$a_n = d_1^n + d_2^n + ... + d_k^n$. Determine $\{p \in N^* | \, (p, a_n) = 1 \text{ for all } n \in N^*\}.$

Mihály Bencze

PP. 16725. Solve the following system: $\begin{cases} \left(\frac{a}{b}\right)^{\log_x\left(a^2b\right)} = \frac{y}{ab^2} \\ \left(\frac{a}{b}\right)^{\log_y\left(a^2b\right)} = \frac{z}{ab^2} \\ \left(\frac{a}{b}\right)^{\log_z\left(a^2b\right)} = \frac{x}{ab^2} \end{cases}$, where $a, b > 0$.

Mihály Bencze

PP. 16726. Determine all functions $f, g : (0, +\infty) \to R$ for which $F, G : (0, +\infty) \to R$ are there primitives, where $x^a F(x) = f^b(x)$ and $x^b G(x) = g^a(x)$ $(a, b \in R)$, for all $x > 0$.

Mihály Bencze

PP. 16727. Solve the following system: $\begin{cases} 2^{\cos x} = \cos 2y + \cos 3z \\ 2^{\cos y} = \cos 2z + \cos 3x \\ 2^{\cos z} = \cos 2x + \cos 3y \end{cases}$.

Mihály Bencze

PP. 16728. In all triangle ABC holds:

1). $\sum \frac{1}{w_a+h_b} \geq \frac{1}{2s} \sum \frac{c}{w_a+h_b} + \frac{\sqrt{2}}{\sqrt{s^2-r^2-4Rr}}$

2). $\sum \frac{1}{m_a+m_b} \geq \frac{1}{s} \sum \frac{a}{m_a+m_b} + \frac{1}{\sqrt{2(s^2-r^2-4Rr)}}$

Mihály Bencze

PP. 16729. Let $ABCD$ be a convex quadrilateral, we construct in exterior the followings $AE \perp AB$, $AE = xAB$, $CF \perp CB$,$CF = yCB$, $CG \perp CD$, $CG = zCD$, $AH \perp AD$, $AH = tAD$. If M, N, P, Q are the midpoints of EB, HD, DG, BF then determine all $x, y, z, t \in R$ for which $MN^2 + PQ^2 = MQ^2 + NP^2$.

Mihály Bencze

PP. 16730. Solve the following system: $\begin{cases} a \log_b(tgx) = \log_a(\sin y) \\ a \log_b(tgy) = \log_a(\sin z) \\ a \log_b(tgz) = \log_a(\sin x) \end{cases}$,

where $a, b, c \in (0, 1) \cup (1, +\infty)$.

Mihály Bencze

PP. 16731. If $(a_n)_{n\geq 1}$ is an increasing natural number sequence, then the sequence $(x_n)_{n\geq 1}$ where $x_n = \sum\limits_{k=1}^{n} \frac{1}{a_1 a_2^2 \ldots a_k^k}$ is convergent and his limit is an irrational number.

Mihály Bencze

PP. 16732. Determine all $a, b, c, d \in Z$ for which $\sum\limits_{k=1}^{n} (ak+b)(ck+d)$ is a perfect square

2). Determine all $a, b, c, d \in Z$ for which $\sum\limits_{k=1}^{n} \frac{ak+b}{ck+d}$ is integer.

Mihály Bencze

PP. 16733. If $x, y, z > 0$ and $x+y+z=1$, then $\sum \frac{1}{x+yz} \geq \frac{3}{2} \sum \frac{1}{x+y}$.

Mihály Bencze

PP. 16734. In quadrilateral $ABCD$ is inscribed an ellipse (E) with center I, and denote (e) a line tangent to the ellipse. Let A_1, B_1, C_1, D_1 be the projection of A, B, C, D onto (e). Determine all ellipse (E) for which $\frac{AA_1 \cdot BB_1}{CC_1 \cdot DD_1} = \frac{AI \cdot BI}{CI \cdot DI}$.

Mihály Bencze

PP. 16735. Let ABC be a triangle inscribed in an ellipse (E), the interior bisector, of $BCA\measuredangle$ meet the ellipse (E) in R, the mediator of BC meet the ellipse (E) in P, and the mediator of AC meet the ellipse (E) in Q. Denote K and L the midpoint of BC and AC. Determine all ellipse (E) for which $Area\,[RPK] = Area\,[RQL]$.

Mihály Bencze

PP. 16736. If $21^{x_n} + 22^{x_{n+1}} + 23^{x_{n+2}} + 24^{x_{n+3}} = 25^{x_{n+4}} + 26^{x_{n+5}} + 27^{x_{n+6}}$ for all $n \in N$, then determine all $x_0, x_1, x_2, x_3, x_4, x_5 \in R$ such that $x_{2010} = 1,9$. If $x_0, x_1, x_2, x_3, x_4, x_5 \in [1, 2)$, then $(x_n)_{n\geq 0}$ is convergent, and compute its limit.

Mihály Bencze

PP. 16737. Solve the equation
$2\left(1^3+2^3+...+n^3\right)+(x-n)^3+...+(x-1)^3+2x^3=(x+1)^3+...+(x+n)^3$,
where $n\in N^*$.

Mihály Bencze

PP. 16738. Let ABC be a triangle inscribed in an ellipse. The median AD meet the ellipse in D_1, the cevian CE meet AD in F and the ellipse in F_1. Suppose $\frac{AE}{EB}=\frac{CF}{FE}=\lambda$, determine $Area\left[AF_1BD_1CA\right]$ in function of λ.

Mihály Bencze

PP. 16739. The triangle ABC is inscribed in an ellipse. Let M be a point on ellipse lying on the arc BC that does not contain A. Let I be the incenter of ABC, and let E and F be the feet of the perpendiculars from I to liner MB and MC, respectively. Determine all ellipses for which $\frac{IE+IF}{AM}$ is independent of the position of M.

Mihály Bencze

PP. 16740. Let be H an interior point in triangle ABC, denote M the midpoint of AB, and H_1, H_2 be the feed of perpendiculars from H to the interior bisector and the exterior bisector of angle $ACB\measuredangle$. Determine all H for which M, H_1, H_2 are collinear.

Mihály Bencze

PP. 16741. Let O be an interior point in triangle ABC. We construct the perpendiculars $OK\perp BC$, $OL\perp AB$, $OM\perp AC$, $k\in BC$, $L\in AB$, $N\in LK$. Determine all points O for which the line BN meet AC in midpoint.

Mihály Bencze

PP. 16742. Let ABC be a triangle inscribed in an ellipse. Denote H the orthocentrum of triangle ABC, and H_1 the projection of H to the interior bisector of angle $ACB\measuredangle$ and H_2 the proiection of H to the exterior bisector of angle $ACB\measuredangle$.
1). Determine all point $M\in AB$ and $N\in AC$ for which M, H_1, H_2, N are collinear
2). Determine all points E on the ellipse such that E, H_1, H_2 are collinear.

Mihály Bencze

PP. 16743. Determine all $z \in C^*$ and all $n, k \in N$ such that
$\left|z^n + z^{-k}\right| = \left|z^k + z^{-n}\right| = 2.$

Mihály Bencze

PP. 16744. The triangle ABC $(AB \neq AC)$ is inscribed in an ellipse. The tangent from A to the ellipse intersect BC in D. The perpendicular in B to BC intersect the mediator of side AB in E, the perpendicular in C to BC intersect the mediator of side AC in F. Prove that D, E, F are collinear.

Mihály Bencze

PP. 16745. If $a_k \in Z$ $(k = 1, 2, ..., n)$ and $d = (a_1, a_2, ..., a_n)$, then solve the following equation: $\sum\limits_{k=1}^{n} \left[\frac{a_k}{x} + \frac{x}{a_k}\right] = nd$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 16746. Determine all $p \in N$ such that

$$\sum_{k=1}^{n} \frac{k(k+1)...(k+p-1)}{p} = \frac{n(n+1)...(n+p)}{(p+1)!} \text{ for all } n \in N^*.$$

Mihály Bencze

PP. 16747. A sequence is generated by listing (from smallest to largest) for each positive integer n the multiples of n up to and including n^k. Denote S_m the m^{th} term of the sequence. Determine all k, m such that:

1). $S_m = \sigma(mk)$ 2). $S_m = F_n^k$

3). $S_m = d(m)\,d(k)$ 4). $S_n = \Psi(mk)$

5). S_m is prime

Mihály Bencze

PP. 16748. Find all functions $f, g : (0, +\infty) \to (0, +\infty)$ such that

$$\begin{cases} f(x + g(y)) = f(x + y) + g(y) \\ g(x + f(y)) = g(x + y) + f(y) \end{cases} \text{ for all } x, y \in R.$$

Mihály Bencze

PP. 16749. Determine all positive integers n for which the numbers in the set $A = \{1, 2, ..., n\}$ can be coloured red, blue, green so that $A \times A \times A \times A$ contains exactly k ordered (x, y, z, t) elements with properties:

a). x, y, z, t are of the same colour
b). $x + y + z + t$ is divisible by n.

Mihály Bencze

PP. 16750. Determine all triangle ABC such that $\frac{a}{h_a} + \frac{b}{w_b} + \frac{c}{m_c} = 2\sqrt{3}$.

Mihály Bencze

PP. 16751. Determine all $a, b, c \in N$ such that $\frac{a+b+1}{c+2}, \frac{b+c+2}{a+3}, \frac{c+a+3}{b+4}$ are integers.

Mihály Bencze

PP. 16752. If $x \in R$, then

$$\left(\frac{1+\sin^4 x}{1+\sin^8 x} + \frac{1+\sin^8 x}{1+\sin^{16} x}\right)\left(\frac{1+\cos^4 x}{1+\cos^8 x} + \frac{1+\cos^8 x}{1+\cos^{16} x}\right) \leq \left(\frac{2}{\sin 2x}\right)^2.$$

Mihály Bencze

PP. 16753. Determine all $\alpha > 0$ such that exist positive integers n and $k > \alpha \cdot 3^n$ for which on can select $3p$ pairwise distinct subsets $A_1, ..., A_p, B_1, ..., B_p, C_1, ..., C_p$ of the set $\{1, 2, ..., n\}$ such that $A_i \cap B_j \cap C_t \neq \emptyset$ for all $1 \leq i, j, t \leq p$.

Mihály Bencze

PP. 16754. A convex $n-$ gon M in the plane is given. For every k $(1 \leq k < n)$ vertices of M, the k- gon determined by then is good if all sides are of unit length. Prove that M has at most $\frac{(k-1)n}{k}$ good k- gons.

Mihály Bencze

PP. 16755. A convex polyhedron P in the space is given. For every six vertices of P the tetrahedron determinated by them is good if all its sides are of unit length. Prove that P has at most $\frac{5n}{6}$ god polyhedrons.

Mihály Bencze

PP. 16756. The diagonals of a convex quadrilateral $ABCD$ intersect at point P. Point Q lies between the lines BC and AD such that $AQD\measuredangle = CQB\measuredangle$, and line CD separate points P and Q. Determine all quadrilaterals $ABCD$ such that $BQP\measuredangle = DAQ\measuredangle$.

Mihály Bencze

PP. 16757. Determine all $k \geq 2$ for which $2\binom{2^{k+2}}{2^{k+1}} - \binom{2^{k+1}}{2^k} - \binom{2^k}{2^{k-1}}$ is divisible by 8^k.

Mihály Bencze

PP. 16758. Find all surjective functions $f, g : N \to N$ such that for every $m, n \in N$ and every prime p, the numbers $f(m+n)$ and $g(m+n)$ are divisible by p if and only if $f(m) + g(n)$ and $f(n) + g(m)$ are divisible by p.

Mihály Bencze

PP. 16759. A circle is divided into $3n$ congruent sectors, n of them coloured black, n of them coloured white, n of them coloured red. Starting with an arbitrary chosen sector, the white sectors are numbered clockwise from 1 to n. Subsequently, the black sectors are numbered counterclockwise from 1 to n, again starting at an arbitrary sector. Subsequently, the red sectors are numbered $a_1, a_2, ..., a_n$. Determine all $a_1, a_2, ..., a_n$ such that there exist n consecutive sectors containing all of the numbers 1 to n.

Mihály Bencze

PP. 16760. Let a be a positive integer. How many nonnegative integers x and k satisfy the equation $k\left[\frac{x}{a}\right] = \left[\frac{x}{a+1}\right] + ... + \left[\frac{x}{a+k}\right]$ when $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 16761. Let n be a positive integer divisible by $k \in N^*$. Find all k and the number of permutations a, b of $\{1, 2, ..., n\}$ which satisfy the condition $a(i) + b(j) + a^{-1}(j) + b^{-1}(i) = 2n + 2$ for all $i, j \in \{1, 2, 3, ..., n\}$.

Mihály Bencze

PP. 16762. In all triangle ABC holds $\sum a^2 + \sum ab \geq$

$$\geq 2\sqrt{2}\left(a\sqrt[4]{a^2(s-b)(s-c)} + b\sqrt[4]{b^2(s-c)(s-a)} + c\sqrt[4]{c^2(s-a)(s-b)}\right).$$

Mihály Bencze

PP. 16763. The positive divisors $d_1, d_2, ..., d_k$ of a positive integer n are ordered $1 = d_1 < d_2 < ... < d_k = n$. Suppose that $d_r^2 + d_{2r+1}^2 = d_{2r+2}^2$ find all possible values of d_{2r+3}.

Mihály Bencze

PP. 16764. If $a, b, c > 0$ then determine all $x \in (0,1)$ such that
$(a+b)(a+c) = \left(a + \left(\frac{b^x+c^x}{2}\right)^{\frac{1}{x}}\right)^2$.

Mihály Bencze

PP. 16765. If $a_0 \in R$ and $a_{i+1} = f(a_i)(a_i - f(a_i))$ for all $i \geq 0$, then determine all functions $f : Z \to Z$ such that $a_i = a_{i+2}$ for sufficiently large i.

Mihály Bencze

PP. 16766. If $a_i > 0$ $(i = 1, 2, ..., n)$ then determine all $k \in N$ such that
$\sum\limits_{1 \leq i_1 < ... < i_k \leq n} \frac{1}{\frac{1}{a_{i_1}} + \frac{1}{a_{i_2}} + ... + \frac{1}{a_{i_k}}} \leq \frac{n}{k\sum\limits_{i=1}^{n} a_i^{k-1}} \sum\limits_{1 \leq i_1 < ... < i_k \leq n} i_1 i_2 ... i_k$.

Mihály Bencze

PP. 16767. Let ABC be a triangle. Determine all $k \in N$ such that
$\sum \frac{\sqrt[k]{b+c-a}}{\sqrt[k]{b} + \sqrt[k]{c} - \sqrt[k]{a}} \leq 3$.

Mihály Bencze

PP. 16768. Denote $S(n)$ the sum of the digits of n. Determine all $k \in N$ for which $S(n) + S(n^2) + ... + S(n^k)$ is a perfect $k-1$ power for infinitely many positive integers n that are not divisible by 10.

Mihály Bencze

PP. 16769. If $a_i > 0$ $(i = 1, 2, ..., n)$, then
$$\sum_{cyclic} \sqrt[k]{\frac{a_1(a_2+...+a_{k+1})}{a_1^k+a_2...a_{k+1}}} \leq \sqrt[k]{\frac{k^2n^{k-2}}{2}\left(\sum_{i=1}^n a_i\right)\sum_{cyclic}\frac{1}{a_1+...+a_k}}.$$

Mihály Bencze

PP. 16770. If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\sum_{k=1}^n a_k = n$, then
$$\sum_{cyclic} a_1 \sqrt[n-2]{a_2...a_n} \leq \frac{n}{2}\left(1 + \sqrt[n-2]{\prod_{k=1}^n a_k}\right).$$

Mihály Bencze

PP. 16771. Let $ABCDE$ be a convex pentagon such that $BAC\measuredangle = xEAD\measuredangle$ and $BCA\measuredangle = yEDA\measuredangle$, and the lines CB and DE intersect in the point F. Determine all $x, y \in R$ such that the midpoints of CD, BE and AF are collinear.

Mihály Bencze

PP. 16772. If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\prod_{k=1}^n a_k = 1$, then
$$\sum_{cyclic} \frac{1}{\sqrt[n-1]{a_1^n+(n-1)a_2^n+n^{n-1}-n}} \leq 1.$$

Mihály Bencze

PP. 16773. In all triangle ABC holds
$$\sum \frac{a^6+b^2c^2}{a^2} \geq \frac{4}{3}\left(2s^2Rr + s^2 + r^2 + 4Rr\right).$$

Mihály Bencze

PP. 16774. If $a_k \in (0, 1)$ $(k = 1, 2, ..., n)$, then
$$\sum_{cyclic} \log_{a_1} \frac{na_2a_3...a_n}{a_2a_3...a_n+a_2^{n-2}+a_3^{n-2}+...+a_n^{n-2}} \geq n - 1.$$

Mihály Bencze

PP. 16775. Solve the following equation $\left\{\sin^k x\right\} + \left\{\cos^k x\right\} = 1$, when $k \in N$ is giving, and $\{\cdot\}$ denote the fractional part.

Mihály Bencze

PP. 16776. Prove that 676^n is the sum of three perfect squares, for all $n \in N^*$.

Mihály Bencze

PP. 16777. Determine all $x \in R$, $k \in Z$ such that:
1). $\sin x + x^k = 0$
2). $\cos x + x^k = 0$.

Mihály Bencze

PP. 16778. Let ABC be a triangle. Determine all points M in space such that $\sum \frac{MA}{MA^2+MB\cdot MC} \leq \frac{1}{2} \sum \frac{1}{MA}$.

Mihály Bencze

PP. 16779. If $a_k, b_k > 0$ $(k = 1, 2, ..., n)$, $A = \sum\limits_{k=1}^{n} a_k$, $B_k = \sum\limits_{k=1}^{n} b_k$, then $\sum\limits_{k=1}^{n} \frac{a_k^2}{b_k} \geq 2 \max \left\{ \frac{a_i(A-a_i)}{\sqrt{b_i(B-b_i)}} | i = 1, 2, ..., n \right\}$.

Mihály Bencze

PP. 16780. The triangle ABC is rectangle if and only if $\frac{s^2-r^2-2Rr}{s^2+r^2+2Rr} \in \left\{ \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \right\}$.

Mihály Bencze

PP. 16781. Solve in N the equation
$x^2 + (y+1)^2 + 4(1+2+...+z)^2 = \left(1+t+u^2\right)^2$.

Mihály Bencze

PP. 16782. Solve in N the equation $\sum\limits_{k=1}^{n} x_k^2 = (2n+1) \sum\limits_{k=1}^{n} x_k$.

Mihály Bencze

PP. 16783. Let ABC be a triangle such that $M \in (BC)$, $N \in (CA)$, $K \in (AB)$. Prove that $\sum \left(\frac{AC}{MB} + \frac{AB}{MC} \right) \frac{BC}{\sqrt{AB \cdot AC}} \geq 12$.

Mihály Bencze

PP. 16784. If $x, y, z, t, u > 0$, then
$2(x+y)(z+t+u)+(x+y+2z)(t+u) \geq 2\left(\frac{3}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{zt}+\frac{1}{zu}+\frac{3}{tu}\right).$

Mihály Bencze

PP. 16785. Determine all $x, y, z > 0$ such that
$\left(\sum\limits_{k=1}^{n} a_k^x\right)\left(\sum\limits_{k=1}^{n}\left(\frac{a_k}{b_k}\right)^y\right)=\left(\sum\limits_{k=1}^{n} a_k^z\right)^2$, where $a_k > 0$ $(k=1,2,...,n)$.

Mihály Bencze

PP. 16786. Solve that following system:
$\begin{cases} \left(a^{2x}+b^{2y}\right)\left(c^{2y}+d^{2z}\right)=\left((ac)^x+(bd)^y\right)^2 \\ \left(a^{2y}+b^{2z}\right)\left(c^{2z}+d^{2x}\right)=\left((ac)^y+(bd)^z\right)^2 \\ \left(a^{2z}+b^{2x}\right)\left(c^{2x}+d^{2y}\right)=\left((ac)^z+(bd)^x\right)^2 \end{cases}$, where $a, b, c, d > 1$ are giving.

Mihály Bencze

PP. 16787. In all triangle ABC holds
$\min\left\{\sqrt{\frac{r}{a}}+\frac{1}{2}\left(\frac{1}{\sqrt{\sin A}}+\frac{1}{\sqrt{\sin B}}\right);\sqrt{\frac{r}{b}}+\frac{1}{2}\left(\frac{1}{\sqrt{\sin B}}+\frac{1}{\sqrt{\sin C}}\right);\sqrt{\frac{r}{c}}+\frac{1}{2}\left(\frac{1}{\sqrt{\sin C}}+\frac{1}{\sqrt{\sin A}}\right)\right\} \geq$
$\sqrt{\frac{(R+r)(s^2+r^2+4Rr)}{4sRr}}.$

Mihály Bencze

PP. 16788. Determine all $A, B \in M_2(R)$ such that
$(tr(A+B))^2 - tr\left(A^2+B^2\right) = 2\det(A+B).$

Mihály Bencze

PP. 16789. If $x, y \in R$, then
$(\sin x)^{2\sin^2 y}+(\sin x)^{2\cos^2 y}+(\cos x)^{2\sin^2 y}+(\cos x)^{2\cos^2 y} \leq 3.$

Mihály Bencze

PP. 16790. If $x \in R$, then $\sin^2 2x + 5(\sin x + \cos x)(2 - \sin x \cos x) \leq 12.$

Mihály Bencze

PP. 16791. Solve in Q the equation $x^5 + y^5 = z^5$.

Mihály Bencze

PP. 16792. In all triangle ABC holds:
1). $\min\left\{\left|\sin B-\cos\frac{A}{2}\right|;\left|\sin C-\cos\frac{A}{2}\right|\right\}\geq\left|\sin A-\cos\frac{B-C}{2}\right|$
2). $\min\left\{\left|\sin B-\sin\frac{A}{2}\right|;\left|\sin C-\sin\frac{A}{2}\right|\right\}\geq\left|\sin A-\cos\frac{A}{2}\right|$.

Mihály Bencze

PP. 16793. In all triangle ABC holds
$\sum\left(xh_a+yw_a+zm_a\right)\log_b c\geq 9\left(x+y+z\right)r$ for all $x,y,z>0$.

Mihály Bencze

PP. 16794. If $x_i>0$ $(i=1,2,...,n)$, then determine all $p,k\in N$ such that
$\left(\prod\limits_{i=1}^{n}x_i\right)^p\left(\sum\limits_{cyclic}\left(\frac{x_1}{x_2}\right)^{n(n-k)}\right)\geq\sum\limits_{i=1}^{n}x_i^n$.

Mihály Bencze

PP. 16795. If $a_i>0$ $(i=1,2,...,n)$ and $P=\prod\limits_{i=1}^{n}a_i$, then
$\left(\sum\limits_{i=1}^{n}a_i\right)\left(\sum\limits_{i=1}^{n}\frac{1}{a_i}\right)\geq\frac{2P}{n^{n-2}}\sum\limits_{i=1}^{n}\frac{1}{P+a_i^n}$.

Mihály Bencze

PP. 16796. If $a,b,c,d\in Z$, then solve the equation $\left[\frac{ax+b}{cx+d}\right]=\frac{a[x]+b}{c[x]+d}$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 16797. If $x>0$ then determine all $n,k\in N^*$ for which
$x+\frac{1}{x}+\left(1+\frac{nx}{1+x^n}\right)^{\frac{1}{k}}\geq 2+2^{\frac{1}{k}}$.

Mihály Bencze

PP. 16798. In all triangle ABC holds
$\max\left\{\sum\frac{h_a}{2h_a^2+h_b^2+h_c^2};\sum\frac{r_a}{2r_a^2+r_b^2+r_c^2}\right\}\leq\frac{1}{4r}$.

Mihály Bencze

PP. 16799. Prove that in all triangle ABC the following inequalities holds
1). $\sum \frac{1}{a^2} \leq \frac{1}{4sr} \sum ctg \frac{A}{2}$
2). $\left(\sum a^2\right)\left(\sum a^4\right) \geq 16s^2r^2\left(2R-r\right)^2$
3). $\sum \frac{1}{a^2} + \sum \frac{1}{ab} \geq \frac{s^2}{2r^2\left((4R+r)^2-2s^2\right)}$
4). $\sum \frac{tg\frac{A}{2}}{\sqrt{bc}} \leq \frac{1}{2r}$

Mihály Bencze

PP. 16800. If $a_k > 1$ $(k = 1, 2, ..., n)$ and $\alpha \geq 1$, then
$\sum\limits_{cyclic} \log_{a_1} \frac{a_2^\alpha + a_3^\alpha + ... + a_n^\alpha}{a_2 + a_3 + ... + a_n} \geq (\alpha - 1)\, n.$

Mihály Bencze

PP. 16801. Determine all function $f : Q \to (0, +\infty)$ such that $f(0) = 1$ and
$f^4(x) = f(x + y + z)\, f(-x + y + z)\, f(x - y + z)\, f(x + y - z)$ for all
$x, y, z \in Q$.

Mihály Bencze

PP. 16802. If $x, y, z > 0$, then
$\max\left\{\sqrt{xy} + \sqrt{xz} - \sqrt{yz}; \sqrt{yz} + \sqrt{yx} - \sqrt{zx}; \sqrt{zx} + \sqrt{zy} - \sqrt{xy}\right\} \leq$
$\frac{1}{2}(x + y + z)$.

Mihály Bencze

PP. 16803. If $x, y > 0$, then $\frac{x^2-xy+y^2}{(x^2+2y^2)(y^2+2x^2)} + \frac{2}{(x+2y)(y+2x)} \geq \frac{1}{x^2+xy+y^2}$.

Mihály Bencze

PP. 16804. If $x \in R$, then $\sin x \cos x\, (\sin x + \cos x) \leq \frac{37}{27}$.

Mihály Bencze

PP. 16805. If $a_k > 1$ $(k = 1, 2, ..., n)$, then solve the following equation:
$$\sum_{1 \leq i < j \leq n} \left(a_i^x a_j^{\frac{1}{x}} + a_i^{\frac{1}{x}} a_j^x\right) + \left(\sqrt{\sum_{k=1}^n a_k^2}\right)^{x+\frac{1}{x}} = \left(\sum_{k=1}^n a_k\right)^2.$$

Mihály Bencze

PP. 16806. If $a_0 = 1$ and $a_{n+1}(1 + a_n) = \sqrt{n}$ for all $n \geq 1$, then compute:
1). $\lim\limits_{n\to\infty} \frac{1}{n^2} \sum\limits_{k=1}^{n} a_k^4$
2). $\lim\limits_{n\to\infty} \sqrt[4]{n} \left(1 - \frac{a_n^4}{n}\right) \in (0, 4)$
3). $\sum\limits_{k=1}^{n} a_k^4 \leq \frac{n^2-n+2}{2} \leq \sum\limits_{k=1}^{n} a_{k^4}$

Mihály Bencze

PP. 16807. Determine all function $f : R \to R$ for which
$\int\limits_1^2 f^2\left(x^2\right) dx + 5\int\limits_2^3 f\left(x^2\right) dx + 7\int\limits_3^4 f(x)\, dx = \frac{1871}{30}$.

Mihály Bencze

PP. 16808. If $x \geq 0$, then
$\left(1 + x^{n+2}\right)^{n+1} \leq (1 + x^n)^{n+1}\left(1 + x^{n+1}\right)^2 \leq 4\left(1 + x^{n+2}\right)^3$.

Mihály Bencze

PP. 16809. If $a, b, c > 0$, then $\sum \frac{a\sqrt{a}}{c+\sqrt{ab}} \geq \frac{\sqrt{3(a+b+c)}}{2}$.

Mihály Bencze

PP. 16810. If $x_k \in R$ $(k = 1, 2, ..., n)$ and $y > 0$ then
$y^{2n} + \frac{2}{n}\left(\sum\limits_{k=1}^{n} \sin x_k\right) y^n + 1 \geq (y - 1)^{2n}$.

Mihály Bencze

PP. 16811. 1). If $x_1 \in R$ and $x_{n+1}\left(1 + x_n^2\right) = 2$ for all $n \geq 1$, then compute $\lim\limits_{n\to\infty} n(x_n - 1)$
2). If $x_1 > 0$ and $x_{n+1}\left(1 + x_n^k\right) = k$ for all $n \geq 1$, where $k > 0$ then compute $\lim\limits_{n\to\infty} x_n$.

Mihály Bencze

PP. 16812. Solve the following equation $[\log_2 x + \log_3 x] + [\log_2 x + \log_3 x + \log_4 x] + ... + [\log_2 x + \log_3 x + ... + \log_n x] = n - 2$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 16813. Determine all function $f : Q \to Q$ such that
$f\left(\prod\limits_{k=1}^{n} x_k + \sum\limits_{k=1}^{n} f(x_k)\right) = \prod\limits_{k=1}^{n} f(x_k) + \sum\limits_{k=1}^{n} x_k$ for all $x_k \in Q$ $(k = 1, 2, ..., n)$
and $f(1) = 1$.

Mihály Bencze

PP. 16814. If $f : [0, +\infty) \to [0, +\infty)$,where $f(x) = \frac{(1+x^{\alpha})^{\beta}}{\left(1+x^{\beta}\right)^{\alpha}}$ $(\alpha, \beta > 0)$, then compute $\max f$, and $\min f$. Determine all $\alpha, \beta > 0$ for which f is bijective.

Mihály Bencze

PP. 16815. In all acute-angle triangle ABC holds $\sum \frac{\sqrt{5a^2-b^2-c^2}}{-m_a+m_b+m_c} \leq 3$.

Mihály Bencze

PP. 16816. If I denote the identric mean then for all $0 < a < b$ holds
$\frac{I\left(a, \frac{a+b}{2}\right)}{I\left(\frac{a+b}{2}, b\right)} \leq e^{\frac{1}{2}\left(1-\frac{a}{b}\right)}$.

Mihály Bencze

PP. 16817. If I denote the identric mean and if $I(x, b) \geq e^{\frac{x+b}{2}}$ for all
$x \in [a, b]$ $(0 < a < b)$, then $\int\limits_{a}^{b} (x \ln a - a)^3 dx \geq \frac{1}{3}(b-a)^3$.

Mihály Bencze

PP. 16818. If in triangle ABC we have the condition $s^2 = 2 + r^2 + 4Rr$,
then $\sum \frac{m_a^3}{m_a m_b + m_c^2} \geq \frac{3}{2}$.

Mihály Bencze

PP. 16819. If $a_k \geq 0$ $(k = 1, 2, ..., n)$ and $\lambda \geq 1$, then

$$\left(\sum_{k=1}^{n} a_k^{\lambda+1}\right)^{\lambda} \leq \left(\sum_{k=1}^{n} a_k^{\lambda}\right)^{\lambda+1} \leq n\left(\sum_{k=1}^{n} a_k^{\lambda+1}\right)^{\lambda}.$$

Mihály Bencze

PP. 16820. If $a_n = n^{\lambda-1} \sum_{k=1}^{n} (|z_k|)^{\lambda} - \left(\left| \sum_{k=1}^{n} z_k \right| \right)^{\lambda}$, when $z_k \in C$ $(k = 1, 2, ..., n)$ and $\lambda \geq 1$, then the sequence $(a_n)_{n \geq 1}$ is increasing.

Mihály Bencze

PP. 16821. If $x, y \in \left(0, \frac{\pi}{2}\right)$, then
$\frac{1}{\sin^2 x \cos^2 x} + \frac{1}{\sin^2 y \cos^2 y} \geq \frac{2}{1-\sin x \sin y} + \frac{2}{1-\cos x \cos y}.$

Mihály Bencze

PP. 16822. If $x, y, z, t, u \in \left(0, \frac{\pi}{2}\right)$, then
$\frac{1}{\left(\sin^2 x+\sin^2 y+\sin^2 z+\sin^2 t+\sin^2 u\right)(\cos^2 x+\cos^2 y+\cos^2 z+\cos^2 t+\cos^2 u)} \geq$
$\frac{2}{25} + \frac{1}{5} \left(\sin^2 x \sin^2 y \sin^2 z \sin^2 t \sin^2 u + \cos^2 x \cos^2 y \cos^2 z \cos^2 t \cos^2 u\right)$

Mihály Bencze

PP. 16823. If $F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n$ for all $n \geq 1$ denote the Fibonacci sequence, then determine all prime p for which $F_n + F_{pn}$ is divisible by p.

Mihály Bencze

PP. 16824. If $x \in [1, 3]$, then $x^{x^2} (4 - x)^{(4-x)^2} \geq 256$.

Mihály Bencze

PP. 16825. If $x_1 = 2$ and $x_{n-1} = 1 + \sqrt{x_n - x_{n-1}}$ for all $n \geq 2$, then
$\left(\prod_{k=1}^{n} x_k - 1 \right)^n \geq n^n \left(\prod_{k=1}^{n} x_k \right)^{n-1}.$

Mihály Bencze

PP. 16826. In all nonisoscelle triangle ABC holds $\prod \frac{h_a - h_b}{r_a - r_b} + 8 \prod \sin^2 \frac{A}{2} = 0.$

Mihály Bencze

PP. 16827. Prove that $\sum_{k=1}^{n} \frac{\left(1+1^2\right)\left(1+2^2\right)...\left(1+k^2\right)}{k!} \geq 2^{n+1} - 2.$

Mihály Bencze

PP. 16828. If $a_i > 0$ $(i = 1, 2, ..., n)$ and $k \in \{2, 3, ..., n-1\}$, then
$\sum\limits_{cyclic} \sqrt[k]{a_1 a_2 ... a_{n-k}} \leq \sqrt[k]{\prod\limits_{i=1}^{n} a_k}$.

Mihály Bencze

PP. 16829. Prove that exists $x_k \in N^*$ $(k = 1, 2, ..., n)$ different natural numbers such that $\frac{\sum\limits_{i=1}^{n} x_k^{\lambda+1}}{\sum\limits_{i=1}^{n} x_k^{\lambda}}$ is integer for all $\lambda \in \{1, 2, ..., m\}$.

Mihály Bencze

PP. 16830. Prove that exist distinct natural numbers $x_1, x_2, ..., x_n$ for which $\sum\limits_{k=1}^{n} x_k^2$ is a perfect $4p + 1$ power and $\sum\limits_{k=1}^{n} x_k^3$ is a perfect square.

Mihály Bencze

PP. 16831. If $a_i, b_i > 0$ $(i = 1, 2, ..., n)$ and $\{a_1, a_2, ..., a_n\} = \{b_1, b_2, ..., b_n\}$, then $(a_1 + a_2 + ... + a_k)^{b_1} (a_2 + a_3 + ... + a_{k+1})^{b_2} ... (a_n + a_1 + ... + a_{k-1})^{b_n} \leq$
$\leq k \prod\limits_{i=1}^{n} b_i^{b_i}$ for all $k \in \{1, 2, ..., n\}$.

Mihály Bencze

PP. 16832. Solve in Z the equations
1). $x^2 + yz + 2z^2 = 1$
2). $x_1^n + x_2 x_3 ... x_n + n x_{n+1}^n = 1$.

Mihály Bencze

PP. 16833. Find the first two digits after the decimal point of the number
$\frac{1}{n^2+1} + \frac{1}{n^2+2} + ... + \frac{1}{2n^2}$, $n \in N^*$.

Mihály Bencze

PP. 16834. If $x \in \left[0, \frac{\pi}{2}\right]$, then
$\frac{2}{1+\sin x \cos x} + \frac{1-\sin x \cos x}{3 \sin^2 x \cos^2 x} \geq \frac{9}{(2 \sin x + \cos x)(2 \cos x + \sin x)}$.

Mihály Bencze

PP. 16835. If $x_1 = 1$ and $x_{n+1} = \left| x_n - \frac{n}{n+1} \right|$ for all $n \geq 1$, then compute $\lim\limits_{n\to\infty} n \prod\limits_{k=1}^{n} x_k$.

Mihály Bencze

PP. 16836. Determine all $2 \leq p < q$ primes, and all $n \in N^*$ for which the first two decimals of numbers p^{n+1} and q^n are equal.

Mihály Bencze

PP. 16837. If $a_k > 0$ $(k = 1, 2, ..., n)$, then $\sum\limits_{k=1}^{n} \frac{a_k}{S-a_k} + \sqrt[n]{\prod\limits_{k=1}^{n} \frac{a_k}{S-a_k}} \geq \frac{n+1}{n-1}$ where $S = \sum\limits_{i=1}^{n} a_k$.

Mihály Bencze

PP. 16838. If $x_n = 1 + \frac{1}{2} + ... + \frac{1}{n}$, then determine all $k \in N^*$ for which $x_{kn} > \left(1 + \frac{1}{kn}\right) x_n$ for all $n \in N^*$.

Mihály Bencze

PP. 16839. Solve in Z the equation $x^n + y^n = nz^{n+1} + 2^{n+1}$.

Mihály Bencze

PP. 16840. Determine all $x, y, z, t \in R$ such that $\sin^2 x \sin^2 y + \cos^2 y \cos^2 z + \cos^2 x \cos^2 t + \sin^2 z \sin^2 t \in (1, 2)$.

Mihály Bencze

PP. 16841. If $x, y, z > 0$, then $\sum \frac{x^2+y^2+z^2+3xy}{x+y} \geq 3(x+y+z)$.

Mihály Bencze

PP. 16842. Determine all $a, b, c \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ for which $a^n + b^k + c^m$ is not divisible by $\overline{abc}$ for all $n, k, m \in N$.

Mihály Bencze

PP. 16843. Determine all $a, b, c > 0$ for which $3\sum \frac{1}{a^2} = \left(\sum \frac{1+bc}{1+a}\right)^2$.

Mihály Bencze

PP. 16844. Solve the following equation:
$\sqrt{\underbrace{2+\sqrt{2+...+\sqrt{2+x}}}_{n-time}} + \sqrt{\underbrace{2-\sqrt{2+...+\sqrt{2+x}}}_{n-time}} = x\sqrt{2}$, where $n \geq 3$.

Mihály Bencze

PP. 16845. In all triangle ABC holds the following inequalities:
1). $\frac{Rr}{s^2+r^2+2Rr} \geq \frac{\left(31r^2-s^2+46Rr\right)^3}{532(s^2+r^2+2Rr)^3}$
2). $54R^2r^4 \geq \left(31Rr - 2\left(s^2+r^2\right)\right)^3$

Mihály Bencze

PP. 16846. If $f : [a, b] \to R$ is two time differentiable function with f' and f'' continuous, and $m = \min\limits_{x\in[a,b]} f''(x)$ and $M = \max\limits_{x\in[a,b]} f''(x)$, then
$\frac{m\left(b^2-a^2\right)}{2} \leq bf'(b) - af'(a) - f(b) + f(a) \leq \frac{M\left(b^2-a^2\right)}{2}$.

Mihály Bencze

PP. 16847. Solve in positive integers the equation
$\prod\limits_{k=1}^{n} (n-1+x_k) = \left(n-1+\sum\limits_{k=1}^{n} x_k\right)^{n-1}$.

Mihály Bencze

PP. 16848. If $a, b, c > 0$, then
$\left(\sum (b+c-a)\left(4a^2+b^2+c^2\right) + 2\sum a^3\right)^2 \leq 32\left(\sum a^4\right)\left(\sum a^2 + \sum ab\right)$.

Mihály Bencze

PP. 16849. Determine all triangle ABC such that $\sum \sin^2 \frac{A}{2} \cos^2 \frac{B-C}{2} \leq \frac{3}{4}$.

Mihály Bencze

PP. 16850. In all triangle ABC holds
$\sum \left(tg\frac{A}{2} + ctg\frac{B}{2}\right)\left(tg\frac{B}{2} + ctg\frac{C}{2}\right) \geq 2\left(\frac{s}{r} + \frac{4R+r}{s}\right)$.

Mihály Bencze

PP. 16851. In all triangle ABC holds
$\sum \frac{3\left(s^2-r^2-4Rr\right)+4(m_b m_c-m_c m_a-m_a m_b)}{3(s^2-r^2-4Rr)+4m_b m_c} \geq \frac{3}{5}$.

Mihály Bencze

PP. 16852. In all triangle ABC holds
1). $\left(\sum x\right)^2 \geq 4\sum yz\cos^2\frac{A}{2}$
2). $\left(\sum x\right)^2 \geq 16s\sum\frac{yz(s-a)}{(b+c)^2}$ for all $x, y, z \in R$.

Mihály Bencze

PP. 16853. Prove that in any triangle ABC the following inequality holds:
$\sum \frac{\left(\sin\frac{A}{2}+\cos\frac{B-C}{2}\right)^2}{2\sin^2\frac{A}{2}+\cos^2\frac{B-C}{2}} \leq 8$.

Mihály Bencze

PP. 16854. 1). Prove that $\sum\limits_{i,j=1}^{n}\frac{i^2j^2}{i+j-1} \geq \frac{n^2(n+1)^2}{4}$
2). Prove that $\sum\limits_{i,j=1}^{n}\frac{i^{k+1}j^{k+1}}{i+j-1} \geq \left(\sum\limits_{i=1}^{n} i^k\right)^2$

Mihály Bencze

PP. 16855. If $a_k, b_k, c_k > 0$ $(k = 1, 2, ..., n)$, then
$\sum\limits_{1\leq i,j,k\leq n}\min\{a_ia_ja_k; b_ib_jb_k; c_ic_jc_k\} \leq \sum\limits_{1\leq i,j,k\leq n}\min\{a_ib_jc_k; a_jb_kc_i; a_kb_ic_j\}$.
What happens when min is replaced by max?

Mihály Bencze

PP. 16856. If $0 < y < x$, then
$\prod\limits_{p=1}^{k}\frac{x^{n_p}-y^{n_p}}{n_p} \leq (x-y)^{k-1}\frac{x^{\sum\limits_{p=1}^{k}n_p-k+2}-y^{\sum\limits_{p=1}^{k}n_p-k+2}}{\sum\limits_{p=1}^{n}n_p-k+2}$.

Mihály Bencze

PP. 16857. If $x \in \left[0, \frac{\pi}{4}\right]$, then
$(n+1)\left(2^{n-1}\left(\cos^{2n}x+\sin^{2n}x\right)+1\right)\cos 2x \geq 2^{n+1}\left(\cos^{2n+2}x-\sin^{2n+2}x\right)$.

Mihály Bencze

PP. 16858. If $a, b, c > 0$ and $a^2 + b^2 + c^2 = 1$, then
$\sum \frac{a}{1+2a(b+c)+3bc} \geq \frac{9}{10(a+b+c)}$.

Mihály Bencze

PP. 16859. If $a, b, c, d > 0$ then $\sum a \left(\frac{1}{b+2c} + \frac{1}{b+2d} + \frac{1}{c+2d} \right) \geq 4$.

Mihály Bencze

PP. 16860. In all triangle ABC holds $\sum \left(\frac{\sin \frac{A}{2}}{\cos \frac{B-C}{2}} \right)^2 \geq \frac{3\left(s^2-4Rr-r^2\right)}{2(s^2+r^2+4Rr)}$.

Mihály Bencze

PP. 16861. In all triangle ABC holds
1). $\sum \sqrt{tg \frac{B}{2}} \cos A \leq \frac{1}{2}\sqrt{\frac{s}{r}}$
2). $\sum m_b m_c \cos A \leq \frac{3\left(s^2-r^2-4Rr\right)}{4}$

Mihály Bencze

PP. 16862. In all triangle ABC holds
1). $\frac{s^2+r^2+10Rr}{4Rr} \geq \sum \frac{(a+c)(b+c)\cos A}{c\sqrt{ab}}$
2). $\frac{s}{2\sqrt{r}} \geq \sum \sqrt{r_a} \cos A$

Mihály Bencze

PP. 16863. Find all continuous functions $f : (0, +\infty) \to (0, +\infty)$ satisfying
$f(x) f(y) f(z) = f(xyz) + f\left(\frac{x}{y}\right) + f\left(\frac{y}{z}\right) + f\left(\frac{z}{x}\right)$ for all $x, y, z > 0$.

Mihály Bencze

PP. 16864. Find all continuous functions $f : (0, +\infty) \to (0, +\infty)$ satisfying
$f\left(x^3\right) + f\left(y^3\right) + f\left(z^3\right) = f(xyz) f\left(\frac{x}{y}\right) f\left(\frac{y}{z}\right) f\left(\frac{z}{x}\right)$.

Mihály Bencze

PP. 16865. If $x, y, z \in R$ then in all triangle ABC holds
$2\sum (y+2x) yz^2 \cos A \leq \left(\sum x^2\right)^2$.

Mihály Bencze

PP. 16866. In all triangle ABC holds $\prod\left(1+\frac{s}{r}tg\frac{A}{2}\right)^{ctg\frac{B}{2}}\geq\left(1+\frac{s^2}{r(4R+r)}\right)^{\frac{s}{r}}$.

Mihály Bencze

PP. 16867. Prove that if $x,y,z\in R$, then in all triangle ABC the following inequality holds $\sum\cos(y-z)\cos A\leq\frac{3}{2}$.

Mihály Bencze

PP. 16868. Prove that in all acute-angle triangle ABC the following inequalities holds:
1). $\sum\frac{1}{\sin^2 A}\geq 2\sum\frac{ctgA}{\sin B}$
2). $\sum\frac{1}{\cos^2 A}\geq 2\sum\frac{1}{\cos A}$

Mihály Bencze

PP. 16869. Prove that in any acute-angle triangle ABC the following inequalities holds:

1). $\frac{1}{\sqrt{2}}\sum\frac{\cos A\sin B}{\cos C}\geq\sum\sqrt{\sin 2BtgC}\cos A$
2). $\sum\frac{1}{\sin 2A}\geq\max\left\{\sum\frac{\sqrt{tgAtgC}\cos A}{\sin C};\sum\frac{\sqrt{ctgActgB}}{\cos C}\right\}$.

Mihály Bencze

PP. 16870. Prove that in all triangle ABC the following inequalities holds:
1). $\sum\sin^2 A\geq\frac{sr}{R^2}\sum ctgA$
2). $\sum\cos^2 A\geq 6\cos A\cos B\cos C$
3). $\sum tg^2 A\geq\frac{4sr}{s^2-(2R+r)^2}\sum\frac{\cos^2 A}{\sin A}$
4). $\sum ctg^2 A\geq\frac{s^2-(2R+r)^2}{sr}\sum\sin A$

Mihály Bencze

PP. 16871. Prove that if $x_k>0$ $(k=1,2,...,n)$ are different, then $\sum\limits_{k=1}^{n}x_k^{\lambda}+2^{-n}\sum\limits_{cyclic}(x_1+x_2)^{\lambda}\geq\frac{2}{\lambda+1}\sum\limits_{cyclic}\frac{x_1^{\lambda+1}-x_2^{\lambda+1}}{x_1-x_2}$ for all $\lambda\geq 1$. What happens if $\lambda\in[0,1)$?

Mihály Bencze

PP. 16872. Prove that in any triangle ABC the following inequality holds
$\sum \frac{ab}{(a+c)(b+c)} \leq \frac{2\left(s^2-r^2-Rr\right)^2}{(s^2+r^2+2Rr)^2} - \frac{3\left(s^2-r^2-4Rr\right)}{s^2+r^2+4Rr}$.

Mihály Bencze

PP. 16873. Prove that if $a \in N^*$, then $k < \sum\limits_{i=1}^{\left(a^k-a\right)n} \frac{1}{an+i} < \ln\left(\frac{na^k+1}{na+1}\right)$ for all $n \in N^*$.

Mihály Bencze

PP. 16874. Prove that if $a, b, c > 0$, then the following inequality holds:
$\sum a^3 + \sum ab\,(a+b) \geq \left(\sum a\sqrt{a}\right)^2$.

Mihály Bencze

PP. 16875. Prove that in all triangle ABC the following inequality holds:
$\sum \frac{r_a^3}{r_a^2+r_ar_b+r_b^2} \geq 4R + r - \frac{2s^2}{4R+r}$.

Mihály Bencze

PP. 16876. In all triangle ABC holds the following inequality
$\sum \frac{m_a+m_b-m_c}{a^2+b^2} \geq \frac{27}{8(m_a+m_b+m_c)}$.

Mihály Bencze

PP. 16877. Prove that if $x, y, z \in R$, then $shx + shy + shz \leq$
$\min\left\{chx\sqrt{1+ch^2ych^2z}; chy\sqrt{1+ch^2zch^2x}; chz\sqrt{1+ch^2xch^2y}\right\}$.

Mihály Bencze

PP. 16878. Prove that in any triangle ABC the following inequality holds:
$\sum \frac{\left(a^2-ab+b^2\right)\left(b^2-bc+c^2\right)}{a+c} \geq \frac{2s\left(s^2-3r^2-2Rr\right)^2}{s^2+r^2+2Rr}$.

Mihály Bencze

PP. 16879. Determine all $k > 0$ such that in all triangle ABC the following inequality holds: $\sum \sin\frac{A}{2} \leq \left(k + \frac{r}{kr}\right)^{\frac{1}{k}}$.

Mihály Bencze

PP. 16880. Prove that in any acute-angled triangle ABC the following inequality holds $\frac{\left(s^2+r^2-4R^2\right)^2-8R(R+r)\left(s^2-(2R+r)^2\right)}{4R^4}+\frac{s^2-(2R+r)^2}{2R^2}\leq 1$.

Mihály Bencze

PP. 16881. Determine all triangle ABC such that
$\sum\sqrt{\frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}}\leq\sqrt{2+\frac{s^2+r^2}{Rr}}$.

Mihály Bencze

PP. 16882. Prove that in all triangle ABC holds

1). $\left(s^2+r^2\right)^2\geq 4R^2r^2+Rr\left(13s^2+5r^2+14Rr\right)$
2). $4R^2\geq 2r^2+s^2+4Rr$

Mihály Bencze

PP. 16883. If $a,b,c>0$ then for all $\lambda>0$ holds
$\sum\left(\lambda+\frac{a}{b}\right)^3\geq 3\lambda^3+3\lambda\sum\frac{\lambda a+c}{b}+\sum\frac{a\sqrt{a}}{b\sqrt{b}}$.

Mihály Bencze

PP. 16884. 1). Determine all prime p,q,r such that $pqr=\overline{ABBA}$, where $A,B\in\{0,1,2,3,4,5,6,7,8,9\}$.
2). Determine all prime p,q,r such that $p^2+q^2+r^2=\overline{ABBA}$

Mihály Bencze

PP. 16885. If p_k denote the k-th prime ($p_1=2$) and
$S_n=\frac{1}{p_1p_2}+\frac{1}{p_2p_3}+...+\frac{1}{p_np_{n+1}}$, then exist $n_0\in N^*$ such that for all $n\geq n_0$ holds $\left[3^{k+1}S_n\right]=3^{k-1}$ for all $k\in N^*$, when $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 16886. If $a,b,c>0$ then for all $\lambda>0$ holds
$\sum\left(\lambda+\frac{a}{b}\right)^2\geq 3\lambda^2+\sum\frac{a+2\lambda c}{b}$.

Mihály Bencze

PP. 16887. If p_k denote the k-th prime $(p_1 = 2)$ and $x_n = \sum\limits_{k=1}^{n} \frac{1}{p_k}$, then compute $\lim\limits_{n\to\infty} \frac{e^{x_n}}{\sqrt{n}}$.

Mihály Bencze

PP. 16888. Determine all $\alpha, \beta \in R$ for which the sequence
$a_n = \sum\limits_{k=1}^{n} \frac{1}{k^\alpha - k^\beta + 1}$ is convergent.

Mihály Bencze

PP. 16889. If $\sqrt{11} > \frac{a+1}{ab}$, where $a, b \in N^*$, then $\sqrt{11} > \frac{a+2}{a(a+1)b}$.

Mihály Bencze

PP. 16890. 1). Prove that the set
$M_k = \left\{n, n+1, n+2, ..., n^k + n^{k-1} + ... + n + k - 1\right\}$, where $k \in N$, $k \geq 3$, contain at least one perfect $k+1$ power
2). Determine all $n, k \in N$ $(k \geq 2)$ for which M_k contain exactly $p \in N^*$ perfect $k+1$ power.

Mihály Bencze

PP. 16891. Let $(x_n)_{n\geq 1}$ be a sequence with the following property: exist $M > 0$ such that $0 < x_{n+1} - \lambda x_n \leq M$ for all $n \in N^*$. Determine all $\lambda > 0$ for which exist $k \in N$ such that the first p digits of x_k are in this order $a_1, a_2, ..., a_p$ where $a_i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $(i = 1, 2, ..., p)$.

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PP. 16892. If $x \in R$, then
$$\min\left\{\frac{\sin^2 2x}{1+\sin^2 x} + \frac{16\left(1+\sin^2 x\right)}{\sin^2 2x}; \frac{\sin^2 2x}{\left(1+\sin^2 x\right)\left(1+\cos^2 x\right)} + \frac{16}{\sin^2 2x}\right\} \geq 24.$$

Mihály Bencze

PP. 16893. Determine all functions $f : R \to R$ such that
$f\left(xyz + f(x) + f(y) + f(z)\right) = f(x) f(y) f(z) + \frac{1}{3}\left(f(x) + f(y) + f(z)\right)$
and $f(1) = 1$ for all $x, y, z \in R$.

Mihály Bencze

PP. 16894. 1). If $x_n = \sum\limits_{k=1}^{n} \frac{1}{k}$, then $\left(\frac{x_{2n}}{x_n}\right)^{2n} > \left(1 + \frac{1}{2n}\right)^{2n}$ for all $n \in N^*$

2). Compute $\lim\limits_{n\to\infty} \left(\frac{x_n}{x_{2n}}\right)^{2n}$

Mihály Bencze

PP. 16895. 1). If $x_1 = 1$ and $x_{n+1} = \frac{1}{n} + \frac{1}{x_n^2}$ for all $n \geq 1$, then
$\sum\limits_{k=1}^{n} \frac{1}{x_k} < \sqrt{2n}$

2). Compute $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n}} \sum\limits_{k=1}^{n} \frac{1}{x_k}$

Mihály Bencze

PP. 16896. If $z \in C$ such that $\left|z + \frac{1}{z}\right| = \sqrt{5}$, then
$\left(\frac{\sqrt{5}-1}{2}\right)^2 \leq |z| \leq \left(\frac{\sqrt{5}+1}{2}\right)^2$.

Mihály Bencze

PP. 16897. Solve the following equation

$13^{30x} + 9^{\frac{1}{x}} = 2 \cdot 13^{15x} \cdot 3^{\frac{1}{x}} + 3818116.$

Mihály Bencze

PP. 16898. Solve in N the equation: $x^y + y^x = (x-y)^{x+y}$.

Mihály Bencze

PP. 16899. If $a, b, c \in C^*$ such that $\sum ab = 0$, then $2\left|\sum \frac{1}{a}\right| \leq \sum \left|\frac{a-1}{a^2}\right|$.

Mihály Bencze

PP. 16900. If $a, b, c \in R^*$ such that

$\sum ab = 0$, then $\frac{1}{3}\sum \frac{1}{a^2b^2} + \sum \frac{a+b}{a^2b^2} \geq \sum \frac{1}{ab}$.

Mihály Bencze

PP. 16901. 1). If $a_k, b_k \in N$ $(k = 1, 2, ..., n)$ such that
$\{a_1, a_2, ..., a_n\} = \{b_1, b_2, ..., b_n\} = \{1, 2, ..., n\}$, then $\sum\limits_{k=1}^{n} \frac{a_k^2}{b_k} \geq \frac{n(n+1)}{2}$
2). Determine all $p \in N$ such that $\sum\limits_{k=1}^{n} \frac{a_k^{p+1}}{b_k^p} \geq \frac{n(n+1)}{2}$

Mihály Bencze

PP. 16902. If $x \in R$, then $29\sin^4 x + 4\sin x + 32 \geq 6\sin^3 x + 32\sin^2 x$.

Mihály Bencze

PP. 16903. In all triangle ABC holds
$\left(\sum a^2\right)\left(\sum \frac{1}{2(b^2+c^2)-a^2}\right) \geq 2\sum \frac{2\left(b^2+c^2\right)-a^2}{4a^2+b^2+c^2}$.

Mihály Bencze

PP. 16904. The triangle ABC is rectangle if and only if the equation
$a^2b^2c^2x^6 + 2abc\left(\sum ac^2 \sin A\right)x^5 + \left(\sum a^4b^2 + 4abc\sum a^2c\sin A\sin B\right)x^4 +$
$+\left(2\sum a^3b^3\sin C + 2abc\sum a^3\sin A + 8a^2b^2c^2\prod \sin A\right)x^3 +$
$\left(\sum a^2b^4 + 4abc\sum b^2\sin A\sin B\right)x^2 +$
$+2abc\left(\sum ab^2\sin A\right)x + a^2b^2c^2 = 0$ have at least one real root.

Mihály Bencze

PP. 16905. Determine all $n \in N$ such that the first 2010 digits after decimal point of the number $\left(2+\sqrt{2}\right)^n$ are 9.

Mihály Bencze

PP. 16906. Let ABC be a triangle, prove that the equation
$a^2b^2c^2x^6 + 2abc\left(\sum ac^2\cos A\right)x^5 + \left(\sum a^4b^2 + 4abc\sum a^2\cos A\cos B\right)x^4 +$
$+\left(2\sum a^3b^3\cos C + 2abc\sum a^3\cos A + 8a^2b^2c^2\prod\cos A\right)x^3 +$
$+\left(4abc\sum b^2x\cos A\cos B + \sum a^2b^4\right)x^2 + 2abc\left(\sum ab^2\cos A\right)x + a^2b^2c^2 = 0$
have no real roots.

Mihály Bencze

PP. 16907. If $x \in R$, then
1). $(\sin x + \cos x)(2 + \sin x\cos x) + 8\sin^2 x\cos^2 x \leq 6$
2). $(\sin x + \cos x)(2 - 3\sin x\cos x) + 2\sin^2 x\cos^2 x \leq 9$

Mihály Bencze

PP. 16908. Let $ABCD$ be a convex quadrilateral with sides a, b, c, d and diagonals e, f. If $(a-d)(b-c) \geq 0$, then the equation
$x^2 - xPer(ABCD) + ef = 0$ have distinct real roots.

Mihály Bencze

PP. 16909. If $A \in M_n(C)$ when $A = \begin{pmatrix} x & 1 & 0 & 0 & ... & 0 & 0 & 0 \\ 1 & x & 1 & 0 & ... & 0 & 0 & 0 \\ 0 & 1 & x & 1 & ... & 0 & 0 & 0 \\ - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & ... & 0 & 1 & x \end{pmatrix}$,
then determine all $x \in R$ such that $\det A = \frac{1}{[x]}\left(\frac{[x]+x}{2}\right)^{x^2} + \frac{1}{[x]}\left(\frac{[x]-x}{2}\right)^{x^2}$,
where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 16910. If $a, b, c > 0$ and $a + b + c = 1$, then
$\sum \left|a^2 - 3a + 4\left(b^2 + c^2\right)\right| \leq \sum (a + 4b)|b - a| + \sum (a + 4c)|c - a|$.

Mihály Bencze

PP. 16911. If $a_k \in [-1, 1]$ $(k = 1, 2, ..., n)$, then
$\sum\limits_{cyclic} \left(\sqrt{\frac{a_1^2+a_2^2}{1+a_1^2a_2^2}} + a_1a_2\right) \leq 2\sum\limits_{k=1}^{n} a_k^2$.

Mihály Bencze

PP. 16912. If $a_i > 0$ $(i = 1, 2, ..., n)$, then $\sum\limits_{cyclic} \frac{a_1}{\sqrt[k]{a_1^k + \left(n^k - 1\right)a_2^k}} \geq 1$ for all $k \in N$, $k \geq 2$.

Mihály Bencze

PP. 16913. Determine all $n, k, p \in N$ for which $n < \sum\limits_{i=1}^{k} \frac{1}{\sqrt[p]{k}} < n + 1$.

Mihály Bencze

PP. 16914. Let ABC be a triangle. Prove that the equation $b^4x^4 +$
$+2\sqrt{2}b^3c\cos\left(\frac{\pi}{4} - A\right)x^3 + 2b^2c^2(1 + \sin 2A)x^2 + 2\sqrt{2}bc^3\cos\left(\frac{\pi}{4} - A\right)x + c^4 = 0$
and his permutations have real roots, than the triangle ABC is rectangle.

Mihály Bencze

PP. 16915. If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\prod\limits_{k=1}^{n} a_k \leq 1$, then
$\sum\limits_{cyclic} \frac{a_1}{a_2^{\alpha+2}+a_3^{\alpha+1}+a_4^{\alpha}} \geq \frac{n}{3}$ for all $\alpha \geq 1$.

Mihály Bencze

PP. 16916. If $a_k \geq 2$ $(k = 1, 2, ..., n)$ and $S_n = \sum\limits_{i=1}^{n} a_i$, then
1). $\sum\limits_{k=1}^{n} \log_{s_n - a_k} a_k^{\alpha} \geq \frac{\alpha n}{n-1}$, for all $\alpha > 0$
2). $\sum\limits_{k=1}^{n} p_k \log_{S_n - a_k} a_k \geq \left(\sum\limits_{k=1}^{n} p_k \right) \log_A B$, where $A = \prod\limits_{cyclic} a_1^{p_2+...+p_n}$ and
$B = \prod\limits_{k=1}^{n} a_k^{p_k}$, where $p_k > 0$ $(k = 1, 2, ..., n)$.

Mihály Bencze

PP. 16917. If $a_k > 0$ $(k = 1, 2, ..., n)$ then $\sum\limits_{cyclic} \frac{a_1}{a_2} \geq \sum\limits_{cyclic} \frac{a_1+a_3+a_4+...+a_n}{a_2+a_3+a_4+...+a_n}$.

Mihály Bencze

PP. 16918. If $x, y, z > 0$, then $3 + \sum \frac{x}{y} + \sum \frac{y}{x} + \sum \frac{xy}{z^2} \geq \frac{3(x+y)(y+z)(z+x)}{xyz}$.

Mihály Bencze

PP. 16919. If $x, y, z > 0$, then $\prod \left(xy + z^2 \right) \geq 4x^2y^2z^2 \left(\sum \frac{x}{y} - 1 \right)$.

Mihály Bencze

PP. 16920. If $n \geq k + 1$, $k \geq 2$, then
1). $\left(\sum\limits_{i=1}^{n} \left(\cos \frac{i\pi}{n} \right)^k \right) \left(\sum\limits_{i=1}^{n} \cos \frac{ki\pi}{n} \right) \geq n^2 2^{-k}$
2). $\left(\sum\limits_{i=1}^{n} \left(\cos \frac{i\pi}{n} \right)^{2k} \right) \left(\sum\limits_{i=1}^{n} \cos^2 \frac{ki\pi}{n} \right) \geq n^2 2^{-2k}$

Mihály Bencze

PP. 16921. If $a, b, c > 0$, then
$\left(\sum a \right) \left(\sum \frac{1}{a} \right) \geq 6 \sum \frac{a}{b+c} \geq 6 + 2 \sum \frac{a}{b+c} \geq \frac{31}{3} - \frac{4 \sum ab}{\sum a^2} \geq 9$.

Mihály Bencze

PP. 16922. If $\alpha \in R\backslash Q$, $n \geq 10^{k+2}$ $(k \in N^*)$, and
$M = \{\{\alpha\};\{2\alpha\};...;\{(n+1)\alpha\}\}$, where $\{\cdot\}$ denote the fractional part, then prove that the set M contain at least two different elements which have the same first k digits, after decimal point.

Mihály Bencze

PP. 16923. If $a_k > 0$ $(k = 1, 2, ..., n)$, then
$\left(\sum\limits_{k=1}^{n} a_k\right)\left(\sum\limits_{k=1}^{n} \frac{1}{a_k}\right) \geq 2n \sum\limits_{cyclic} \frac{a_1}{a_2+a_3}$.

Mihály Bencze

PP. 16924. If $a_i > 1$ $(i = 1, 2, ..., n)$ and $k \in \{3, 4, ..., n\}$, then
$$\frac{\prod\limits_{cyclic}\left(\log_{a_2} a_1 \log_{a_3} a_1 ... \log_{a_{k-1}} a_1 + \log_{a_3} a_1 \log_{a_4} a_1 ... \log_{a_k} a_1\right)^{k-2}}{\left(\prod\limits_{i=1}^{n} \log_{a_{i+1}} a_i \log_{a_{i+2}} a_1 ... \log_{a_{i+k-1}} a_i\right)^{k-3}} \geq$$
$$\geq (k-1)^{n(k-2)} \prod_{cyclic} \log_{\frac{a_2+a_3+...+a_k}{k-1}} a_1.$$

Mihály Bencze

PP. 16925. If $a, b, c > 0$, then $3 + \sum \frac{a+b}{c} \geq 6 \sum \frac{a}{b+c}$.

Mihály Bencze

PP. 16926. If $a_k > 0$ $(k = 1, 2, ..., n)$, then $2n(n-3)\sum\limits_{k=1}^{n} a_k^3 \geq$
$$\geq 4\left(\sum_{k=1}^{n} a_k\right)\left(\sum_{k=1}^{n} a_k^2 - \sum_{1 \leq i < j \leq n} a_i a_j\right) + \sum_{1 \leq i < j < k \leq n} (a_i + a_j - 2a_k)^3.$$

Mihály Bencze

PP. 16927. In all triangle ABC holds
$\sum \left(a^2 + b^2 + 4c^2\right) m_a m_b (m_a + m_b) \geq 8 \sum (m_a + m_b) m_a^2 m_b^2$.

Mihály Bencze

PP. 16928. Prove that
$\sum\limits_{k=1}^{n}\left(\sqrt{1+k^3} + \sqrt{1+\sqrt{k^9}} + \sqrt{1+\sqrt{k^3}}\right) \leq 3n + \frac{n(n+1)\left(3n^2+7n+8\right)}{24}$.

Mihály Bencze

PP. 16929. If $x \in R$, then $\left(1+\sin^2 x\right)^{12}+\left(1+\cos^2 x\right)^{12}+\left(1+\sin^2 x\right)^4+$

$+\left(1+\cos^2 x\right)^4 > \frac{5}{2}+\left(1+\sin^2 x\right)^9+\left(1+\cos^2 x\right)^9.$

Mihály Bencze

PP. 16930. If $a_k \geq 5$ $(k=1,2,...,n)$ are prime, then the Vandermonde determinant $V\left(a_1,a_2,...,a_n\right)$ is divisible by $9 \cdot 2^{\frac{n(n-1)}{2}}$.

Mihály Bencze

PP. 16931. Solve in N the equation $x^2+3x^5+5x^7=9x^{14}$.

Mihály Bencze

PP. 16932. If $x \in R$, then:
1). $4+9\sin^4 x\cos^4 x+3\left(\sin x+\cos x\right)\left(1-\sin x\cos x\right)\sin^3 x\cos^3 x>$
$>\sin x+\cos x+8\sin^2 x\cos^2 x+2\sin^6 x\cos^6 x+\left(\sin x+\cos x\right)^3\left(1-\sin x\cos x\right)^3$
2). $4>8\sin^2 x\cos^2 x+3\sin^4 x\cos^4 x+20\sin^6 x\cos^6 x+$
$+\left(\sin x+\cos x\right)\left(2-\sin x\cos x-3\sin^2 x\cos^2 x+12\sin^3 x\cos^3 x+\sin^4 x\cos^4 x\right).$

Mihály Bencze

PP. 16933. Prove that π is not an integer, where π is the ratio of the circumference of a circle to the diameter.

George J. Miliakos

PP. 16934. 1). If $a>1$, then solve the following equation
$\sum_{k=1}^{n} a^{x_k^2}=\sum_{cyclic} a^{\left(\frac{x_1+x_2}{2}\right)^2}$
2). What happens if $0<a<1$?

Mihály Bencze

PP. 16935. Solve the following equation $\sum_{k=1}^{n} a^{x_k^2}=\sum_{cyclic} a^{\left|x_1x_2\right|}$, where $a>0$.

Mihály Bencze

PP. 16936. If $a, b, c > 0$, then
$4a^3 + b^3 + c^3 + 2a\left(b^2 + c^2\right) \geq \left(4a^2 + bc\right)(b + c)$.

Mihály Bencze

PP. 16937. Prove that if the equation $x^3 + ax^2 + bx + c = 0$, have all roots non-negative real numbers, then $\sum (x_2 + x_3 - 2x_1)^3 \leq 12\left(-a^3 + 3ab\right)$. Is the reciprocal affirmation true?

Mihály Bencze

PP. 16938. If $\alpha, \beta \in C \backslash R$, $|\alpha| < 1, |\beta| < 1$, and $k, n \in N$ $(n, k \geq 3)$, then the equation $z^{n+k} + (\alpha + \beta) z^{n+k-1} + \alpha\beta z^{n+k-2} + \overline{\alpha} z^{k+1} + \overline{\beta} z^{n+1} +$
$+ \left(1 + \alpha\overline{\beta}\right) z^n + (1 + \overline{\alpha}\beta) z^k + \alpha z^{n-1} + \beta z^{k-1} + \overline{\alpha\beta} z^2 + \left(\overline{\alpha} + \overline{\beta}\right) z + 1 = 0$
have all roots of modul 1.

Mihály Bencze

PP. 16939. Solve the following equation $\left(1 - ntg^{n-1}x\right) tgnx = ntgx - tg^n x$, when $n \in N$.

Mihály Bencze

PP. 16940. If $x, y, z > 0$ and $xyz = 1$, then $\sum xy^2 \geq \sum x$.

Mihály Bencze

PP. 16941. If $a_k > 0$ $(k = 1, 2, ..., n)$, then

1). $\sum\limits_{k=1}^{n} \sqrt{a_k \left(a_k^3 + (a_1 + ... + a_{k-1} + a_{k+1} + ... + a_n)^3\right)} \leq \frac{n^2 - 2n + 3}{2} \sum\limits_{k=1}^{n} a_k^2$

2). $\sum\limits_{k=1}^{n} \frac{2a_k^2 + (n-1)\left(a_1^2 + ... + a_{k-1}^2 + a_{k+1}^2 + ... + a_n^2\right)}{\sqrt{a_k^3 + (a_1 + ... + a_{k-1} + a_{k+1} + ... + a_n)^3}} \leq 2 \sum\limits_{k=1}^{n} \sqrt{a_k}$

Mihály Bencze

PP. 16942. Determine all $a_i > 0$ $(i = 1, 2, ..., n)$ such that
$(k - 1) \sum\limits_{i=1}^{n} \frac{1}{1 + a_i^k} = \frac{k}{1 + \prod\limits_{i=1}^{n} a_i}$, where $k \in \{2, 3, ..., n - 1\}$.

Mihály Bencze

PP. 16943. 1). In all triangle ABC holds $\sum \frac{1}{\frac{1}{3}+tg^2\frac{A}{2}} \geq \frac{9}{2}$
2). Determine all $x > 0$ such that $\sum \frac{1}{x+tg^2\frac{A}{2}} \geq \frac{3}{x+1}$.

Mihály Bencze

PP. 16944. If $x \geq 2$, then $2x+1 < x^2 \sin\frac{2\pi}{x} + (x+1)^2 \sin\frac{2\pi}{x+1}$.

Mihály Bencze

PP. 16945. If $x \in \left[0, \frac{\pi}{2}\right]$, then $\frac{(2+tgx)^3+(2+ctgx)^3}{\sin x+\cos x} \geq 64\left(1-\sin x\cos x\right)$.

Mihály Bencze

PP. 16946. If $a_k > 1$ $(k = 1, 2, ..., n)$, then
$\prod\limits_{cyclic}\left(\log_{a_2} a_1 + \log_{a_3} a_1\right) \geq 2^n \prod\limits_{cyclic} \log_{\frac{a_2+a_3}{2}} a_1$

Mihály Bencze

PP. 16947. If $f, g : [2, +\infty) \to R$, where $f(x) = x\sin\left(\frac{\pi}{x} - \frac{\pi}{2}\right)$ and
$g(x) = x^2\left(1-\sin\frac{2\pi}{x}\right)$, then prove that f and g are decreasing.

Mihály Bencze

PP. 16948. If $x \in R$, then
$2\sin^4 x + 4tg^2x < 4\sin\left(\pi\sin^2 x\right) + \left(2+\sin^2 x\right)^2 \sin\left(\frac{2\pi\sin^2 x}{2+\sin^2 x}\right)$.

Mihály Bencze

PP. 16949. Determine all $k, m \in N$ for which exist infinitely many positive integers n such that $n^k + n^m$ dividet $n!$

Mihály Bencze

PP. 16950. In all triangle ABC holds

$$3 + \frac{4R}{s}\sum \frac{\sin^2\frac{A}{2}\sin^2\frac{B-C}{2}+\sin^2\frac{B}{2}\sin^2\frac{C-A}{2}}{\cos\frac{A-B}{2}\cos\frac{C}{2}} \leq \frac{4\left(s^2-8Rr-2r^2\right)}{s^2}\sum\frac{\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{A-B}{2}}.$$

Mihály Bencze

PP. 16951. If $x \in R$, then

$\min \left\{ \sum\limits_{n=1}^{\infty} \left(\left\{ n^2 x \right\} - \{x\} + 1 \right); \sum\limits_{n=1}^{\infty} \left(1 + \{x\} - \left\{ n^2 x \right\} \right) \right\} \geq \frac{\pi^2}{6}$, where $\{\cdot\}$ denote the fractional part.

Mihály Bencze

PP. 16952. Determine all $a_k \in Q^*$ $(k = 1, 2, 3, 4, 5, 6)$ for which the function $f : N^* \times N^* \to N^*$ where $f(x, y) = a_1 x^2 + a_2 y^2 + a_3 xy + a_4 x + a_5 y + a_6$ is bijective.

Mihály Bencze

PP. 16953. 1). Prove that in all triangle ABC holds $\left| \frac{2R+r-s}{2R+r+s} \right| \leq 26 - 15\sqrt{3}$
2). Prove that in all acute-triangle ABC holds $\frac{3s^2+5r^2-12R^2}{5s^2+3r^2-12R^2} \leq \frac{1}{27}$.

Mihály Bencze

PP. 16954. Prove that in all triangle ABC holds $\prod \frac{1-tg\frac{A}{4}}{1+tg\frac{A}{4}} \leq \frac{1}{3\sqrt{3}}$.

Mihály Bencze

PP. 16955. If $x \in \left[0, \frac{\pi}{2}\right]$, then $\frac{(2+tgx)^4+(2+ctgx)^4}{1-2\sin^2 x \cos^2 x} \geq 256$.

Mihály Bencze

PP. 16956. If $a_k, b_k > 0$ $(k = 1, 2, ..., n)$ and $\{a_1, a_2, ..., a_n\} = \{b_1, b_2, ..., b_n\}$, then $\left(\prod\limits_{k=1}^{n} a_k^{b_k} + \prod\limits_{k=1}^{n} b_k^{a_k} \right) \left(\sum\limits_{k=1}^{n} a_k \right) \leq 2 \sum\limits_{k=1}^{n} a_k b_k$.

Mihály Bencze

PP. 16957. In all triangle ABC holds:
1). $\sum \frac{1}{\sin^2 A (s-R\sin A)^2} \geq \frac{9R}{2s^2 r}$
2). $\sum \frac{1}{tg^2 \frac{A}{2} \left(4R+r-stg\frac{A}{2}\right)^2} \geq \frac{9}{4r(4R+r)}$
3). $\sum \frac{1}{ctg^2 \frac{A}{2} \left(s-rctg\frac{A}{2}\right)^2} \geq \frac{9}{s^2}$

Mihály Bencze

PP. 16958. If $b > a > 0$, then $\frac{1}{b-a}\int\limits_a^b \left(\frac{x^6+x^3+1}{x^4+1}\right)^2 dx \le$
$\le \frac{9}{8}\left(1+\frac{1}{5}\left(a^2+b^2-\frac{ab(\sqrt{5}-1)}{2}\right)\left(a^2+b^2+\frac{ab(\sqrt{5}+1)}{2}\right)\right).$

Mihály Bencze

PP. 16959. If $a, b > 0$, then
$\left(\frac{a^3+b^3}{a^2+b^2}\right)^{12} \ge \left(\frac{a^3+b^3}{a+b}\right)^6 \ge \left(\frac{a^4+b^4}{2}\right)^3 \ge \left(\frac{a^6+a^3b^3+b^6}{3}\right)^2.$

Mihály Bencze

PP. 16960. If $a, b, c > 0$, then $\sum \frac{1}{a^2+2bc} \le \frac{27(\sum ab)\left(2\sum a^2+\sum ab\right)}{(\sum a)^6}.$

Mihály Bencze

PP. 16961. In all acute triangle ABC holds $\sum (b-c)^2 \cos A \le \sum \frac{a^2(b+c)^2}{b^2+c^2}.$

Mihály Bencze

PP. 16962. In all triangle ABC holds
$\frac{1}{2}\sum \sqrt{5a^2+4m_a^2} \ge \frac{1}{\sqrt{2}}\sum \sqrt{a^2+4m_a^2} \ge \sum \frac{(b+c)^2}{\sqrt{5a^2+4m_a^a}}.$

Mihály Bencze

PP. 16963. If $x, y, z > 0$, then $\sum \sqrt{\frac{x^2+yz}{x(x+y)}} \ge 3.$

Mihály Bencze

PP. 16964. If $\alpha, x_k > 0$ $(k = 1,2,3,4,5,6,7,8)$, then
$\left(\left(\frac{1}{(\alpha+x_1)^2}+\frac{1}{(\alpha+x_2)^2}\right)^2+\left(\frac{1}{(\alpha+x_3)^2}+\frac{1}{(\alpha+x_4)^2}\right)^2\right)^2+$
$+\left(\left(\frac{1}{(\alpha+x_5)^2}+\frac{1}{(\alpha+x_6)^2}\right)^2+\left(\frac{1}{(\alpha+x_7)^2}+\frac{1}{(\alpha+x_8)^2}\right)^2\right)^2 \ge \frac{1}{\alpha^8+x_1x_2x_3x_4x_5x_6x_7x_8}.$

Mihály Bencze

PP. 16965. If $x_k > 0$ $(k = 1,2,...,n)$, then
$x_1^{n-1}+x_2^{n-1}+...+x_n^{n-1} \ge x_1x_2...x_{n-1}+x_2x_3...x_n+...+x_nx_1...x_{n-2}.$

Mihály Bencze

PP. 16966. If $a, b, c > 0$, then $\sum \frac{(a+b)^2(a+c)^2}{a^2+bc} \geq 8\left(ab+bc+ca\right).$

Mihály Bencze

PP. 16967. If $\alpha, x, y, z, t > 0$, then
$\left(\frac{1}{(\alpha+x)^2}+\frac{1}{(\alpha+y)^2}\right)^2+\left(\frac{1}{(\alpha+z)^2}+\frac{1}{(\alpha+t)^2}\right)^2 \geq \frac{1}{\alpha^4+xyzt}.$

Mihály Bencze

PP. 16968. If $a, b, c > 0$, then $\frac{1}{a+b+c}+\frac{2}{\sqrt{ab+bc+ca}} \leq \sum \frac{1}{a+b}.$

Mihály Bencze

PP. 16969. If $a, b, c > 0$ and $(a-b)(a-c) \geq 0$, then
$\left(b^2+c^2\right)a+bc\left(b-c\right)^2 \geq \left(b+c\right)\left(b^2-3bc+c^2\right)a.$

Mihály Bencze

PP. 16970. If $x, y, z, t, u > 0$ and $(y-z)(z-t) \geq 0$, then
$(x+y+z+t+u)^2 \geq 8\left(xz+yt+zu\right)+\left(x+y-3z+t+u\right)^2.$

Mihály Bencze

PP. 16971. If $a, b, c > 0$ and $a+b+c=3$, then
$\sum\left(\frac{1}{a^2-a+3}+\frac{4}{11}\left(a-1\right)^2\left(3-a\right)\right) \leq 1.$

Mihály Bencze

PP. 16972. If $a_k > 0$ $(k=1,2,...,n)$ and $\sum\limits_{k=1}^{n}\left(1+a_k\right)^6=1$, then
$8\sum\limits_{k=1}^{n}\left(1+a_k^2\right)^3 \geq 1+4\sum\limits_{k=1}^{n}\left(1-a_k\right)^4\left(1+a_k+a_k^2\right).$

Mihály Bencze

PP. 16973. If $a_k > 0$ $(k=1,2,...,n)$ and $\sum\limits_{k=1}^{n}\left(1+a_k\right)^4=1$, then
$16\sum\limits_{k=1}^{n}\left(1-a_k+a_k^2\right)^2 \geq 1+8\sum\limits_{k=1}^{n}\left(1-a_k\right)^4.$

Mihály Bencze

PP. 16974. If $a,b,c,d>0$, then $6\sum a^2+\left(\sum a\right)^2+$
$+\frac{1}{3}\left(|2a-b-c+2d|+\sqrt{3}\,|b-c|+\sqrt{3}\,|a-b+c-d|\right)^2\geq 12\left(ab+bc+cd\right).$

Mihály Bencze

PP. 16975. 1). In all triangle ABC holds $\sum\frac{tg\frac{A}{2}}{\frac{2}{\sqrt{3}}+tg\frac{B}{2}}\leq 1$

2). Determine all $\alpha>0$ such that $\sum\frac{tg\frac{A}{2}}{\alpha+tg\frac{B}{2}}\leq 1$

Mihály Bencze

PP. 16976. If $x_k>0$ $(k=1,2,...,n)$ and $\sum\limits_{k=1}^{n}\frac{1}{1+x_k}=\sqrt{2}$, then
$\sum\limits_{k=1}^{n}\frac{1}{\sqrt{1+x_k^2}}\leq 2.$

Mihály Bencze

PP. 16977. 1). If $x,y,z>0$, then $\sum\frac{1}{(x+y+z)^3+54xy^2}\geq\frac{1}{(x+y+z)^3}$
2). Determine all $\alpha>0$ such that $\sum\frac{1}{(x+y+z)^3+\alpha xy^2}\geq\frac{1}{(x+y+z)^3}$

Mihály Bencze

PP. 16978. If $a,b,c>0$, then
$\frac{a+3b}{ab(a+b)}+\frac{b+3c}{bc(b+c)}+\frac{1}{2}\left(\frac{1}{a^2}+\frac{1}{c^2}\right)+\frac{2}{c(a+c)}\geq\frac{4}{a(b+c)}+\frac{4}{ac+b^2}.$

Mihály Bencze

PP. 16979. If $n\geq 4$, then
$\frac{13}{24}<\left(\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{2n}\right)\left(\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{3n}\right)<1.$

Mihály Bencze

PP. 16980. If $n\geq k+1$ $(k\in N^*)$, then $\prod\limits_{k=1}^{m}\left(\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{kn}\right)<\frac{m+1}{2}.$

Mihály Bencze

PP. 16981. If $x\in\left(0,\frac{\pi}{2}\right)$, then $\frac{\sin^8x+ctg^8x}{\cos^6x}+\frac{\cos^8x+tg^8x}{\sin^6x}\geq 5.$

Mihály Bencze

PP. 16982. Determine all $k > 0$ for which $\log_n (n + k) > \log_{n+1} (n + k + 1)$ for all $n \geq 2$.

Mihály Bencze

PP. 16983. If F_n denote the n-th Fibonacci number, then

$\frac{F_n F_{n+5}}{F_{n+1}} + \frac{F_n F_{n+1}}{F_{n+2}} + \frac{F_{n+1} F_{n+3}}{F_{n+5}} + \frac{F_{n+3} F_{n+5}}{F_n} \geq F_{n+6}$ for all $n \geq 1$.

Mihály Bencze

PP. 16984. Determine all $n \in N$ such that $2^n + 3^n + 7^n$ is divisible by 237.

Mihály Bencze

PP. 16985. Determine all $k \in N$ for which $k \sum_{i=1}^{n} i^{k+2}$ is divisible by $\sum_{i=1}^{n} i^k$ for all $n \in N^*$.

Mihály Bencze

PP. 16986. Determine all $n \in N$ such that $5^n - 5, 7^n - 7, 13^n - 13$ are divisible by n.

Mihály Bencze

PP. 16987. Find all integers n such that $n^k - k$ is divisible by $n - k$ for all $k \in N$.

Mihály Bencze

PP. 16988. Determine all positive integers n such that $8n^4 + 1$ is divisible by 5 and by 7.

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PP. 16989. Prove that $27^{n+2} - 338n^2 - 1040n - 729$ is divisible by 494 for all $n \in N^*$.

Mihály Bencze

PP. 16990. If $x_k > 0$ $(k = 1, 2, ..., n)$ and $\sum\limits_{cyclic} \frac{1}{x_1 x_2} = 3$, then
$\sum\limits_{cyclic} \frac{x_1^2 + x_2^2}{x_1^4 + x_1^2 x_2^2 + x_2^4} \leq 2$.

Mihály Bencze

PP. 16991. In all triangle ABC holds $(R + r)(4R + r) \geq s^2$.

Mihály Bencze

PP. 16992. In all triangle ABC holds $\sum \frac{m_a m_b}{5a^2 - b^2 + 8c^2} \leq \frac{3}{16}$.

Mihály Bencze

PP. 16993. If $x, y, z > 0$, then $\sum \frac{y}{x^2 z^2 + 3y^4} \leq \frac{3}{4xyz}$.

Mihály Bencze

PP. 16994. In all triangle ABC holds $\frac{1}{s^2 - r^2 - 4Rr} \sum a^2 A \in \left(\frac{2}{3}; 1\right)$.

Mihály Bencze

PP. 16995. Let ABC be a triangle. Denote R_A the radius of a circle tangent to the sides AB and AC and interior tangent of *arc* $\overset{\frown}{BC}$ in circumcircle of tangent ABC etc. Prove that:

1). $\prod a^{\frac{1}{R_A}} \leq \left(\frac{2s(R+r)}{4R+r}\right)^{\frac{2}{r} + \frac{1}{2R}}$

2). $\left(\sum \frac{\sqrt{a}}{R_A}\right)^2 \leq \frac{s(4R+r)(R+r)}{2R^2 r^2}$

3). $\left(\sum \frac{1}{\sqrt[3]{R_A}}\right)^3 \leq \frac{2(R+r)(4R+r)^2}{Rr}$

Mihály Bencze

PP. 16996. In all triangle ABC holds $\left(\sum \sqrt{a}\right)^2 \leq \frac{2(4R+r)^2}{s}$.

Mihály Bencze

PP. 16997. In all triangle ABC holds:

1). $\frac{1}{2r(4R+r)} \sum \frac{bc - a^2}{3a^2 + b^2 + c^2} \geq \sum \left(\frac{a - b}{2a^2 + 2b^2 + c^2}\right)^2$

2). $\frac{1}{8s^2 r^2} \sum \frac{a^4 - b^2 c^2}{3a^4 + b^4 + c^4} \geq \sum \left(\frac{a^2 - b^2}{2a^4 + 2b^4 + c^4}\right)^2$

Mihály Bencze

PP. 16998. If $a_k > 0$ $(k = 1, 2, ..., n)$, then $\sum\limits_{cyclic} \frac{(a_2+a_3+...+a_n)^2}{(n-1)a_1^2+a_2^2+a_3^2+...+a_n^2} \leq n.$

Mihály Bencze

PP. 16999. If $a, b, c > 0$, then $\prod \left(a^2 + bc\right) \leq \left(\frac{a+b+c}{2}\right)^6$.

Mihály Bencze

PP. 17000. Prove that $\frac{1}{\sqrt{\pi}} < \int\limits_0^{\sqrt{\frac{\pi}{2}}} \cos\left(\frac{\pi}{2} - x^2\right) dx < \sqrt{\frac{\pi}{2}}.$

Mihály Bencze

PP. 17001. If $x, y, z > 0$ and $[\cdot]$ respective $\{\cdot\}$ denote the integer and fractional part, then $\sum \left(\frac{\{x\}^2}{y^2} + \frac{[x]^2}{z^2}\right) \geq \sum \frac{x}{y+z}$.

Mihály Bencze

PP. 17002. If $x, y, z > 0$, then $\sum \frac{1}{x+y} \left(\left(\frac{\sin^2 x}{\sqrt{x}}\right)^2 + \left(\frac{\cos^2 x}{\sqrt{y}}\right)^2\right) \geq \frac{9}{4\sum xy}$.

Mihály Bencze

PP. 17003. If $x, y, z > 0$ and $[\cdot]$ respective $\{\cdot\}$ denote the integer and fractional part, then $\sum\limits_{cyclic} \left(\frac{\{x\}^2}{y^2} + \frac{[x]^2}{z^2}\right) \geq \frac{3\sum x^3}{2\sum x^2}$.

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PP. 17004. If $x, y, z, t > 0$ and $[\cdot]$ respective $\{\cdot\}$ denote the integer and fractional part, then $\sum \frac{1}{x+y} \left(\frac{\{x\}^2}{x} + \frac{[x]^2}{y}\right) \geq 1.$

Mihály Bencze

PP. 17005. In all triangle ABC holds

$$\sum \sqrt{\frac{\sin^4 \frac{A}{2} + \sin^2 \frac{B}{2} \sin^2 \frac{C}{2}}{\sin^4 \frac{B}{2} + \sin^4 \frac{C}{2}}} \geq \frac{1}{\sqrt{2}} + 2\sqrt{\frac{8R^2+r^2-s^2}{s^2+r^2-8Rr}}.$$

Mihály Bencze

PP. 17006. If $0 \leq a_k \leq 1$ $(k = 1, 2, ..., n)$, then
$2 \sum\limits_{k=1}^{n} a_k \geq \sum a_1 a_2 + \sum \frac{a_1^2 + a_2^2}{1 + a_1^2 a_2^2}$.

Mihály Bencze

PP. 17007. In all triangle ABC holds:
1). $\sum \frac{ctg\frac{A}{2}}{\sqrt{3r^2 ctg^2 \frac{A}{2} - 4rs ctg\frac{A}{2} + 2s^2}} \geq \frac{1}{r}$
2). $\sum \frac{1}{\sqrt{2r_a^2 - 2rr_a + 3r^2}} \geq \frac{1}{r}$

Mihály Bencze

PP. 17008. If $x, y, z \in \left[0, \frac{\pi}{2}\right]$, then

$$\sum \sin x \sin y + \sqrt{\frac{\sum \sin^2 x + \sin^2 x \sin^2 y \sin^2 z}{1 + \sum \sin^2 x \sin^2 y}} \leq \sum \sin x + \sin x \sin y \sin z.$$

Mihály Bencze

PP. 17009. If p_k denote the k-th prime, then $\sum\limits_{k=1}^{n} p_k > 66 + \frac{3n(n+1)}{2}$.

Mihály Bencze

PP. 17010. Solve in Z the equation $x^{2n} - x + 1 = y^n$, where $n \in N$.

Mihály Bencze

PP. 17011. Prove that $\sum\limits_{k=1}^{n} \frac{k}{9n^4 + 6n^3 + n^2 - 4k^3} \leq \frac{1}{12n^2}$.

Mihály Bencze

PP. 17012. In all triangle ABC holds
$2 \sum \frac{a^2}{r_a} \geq 4(R + 3r) + (4R + r)\left(\prod a^{2r_a}\right)^{\frac{1}{4R+r}}$.

Mihály Bencze

PP. 17013. If $x, y, z > 0$, then $\sum \frac{1}{xy + 2yz + 2zx + 4z^2} \geq \frac{1}{\sum xy}$.

Mihály Bencze

PP. 17014. If $0 \leq a_k \leq 1$ $(k = 1, 2, ..., n)$ such that $\sum\limits_{cyclic} a_1 a_2 = \frac{1}{2}$, then

$\sum\limits_{cyclic} \frac{a_1+a_2}{(\sqrt{a_1}-\sqrt{a_1 a_2}+\sqrt{a_2})^2} \leq \frac{2n+1}{2}$.

Mihály Bencze

PP. 17015. In all triangle ABC holds $\sum \frac{2\left(b^2+c^2\right)-a^2}{b^2+c^2-a^2} \geq \frac{3}{R} \sum \frac{a^2+b^2}{2(b^2+c^2)-a^2}$.

Mihály Bencze

PP. 17016. Let ABC be a triangle, $M \in Int\,(ABC)$, and d_a, d_b, d_c denote the distance from M to the sides BC, CA, AB. If $\alpha \in [0, 1]$, then $d_a^\alpha + d_b^\alpha + d_c^\alpha \leq 3^{1-\alpha} \left(\frac{s^2+r^2+4Rr-4sr}{2R}\right)^\alpha$.

Mihály Bencze

PP. 17017. Let ABC be a triangle in which $\{D\} = AO \cap BC$, $\{E\} = BO \cap CA$, $\{F\} = CO \cap AB$. Prove that $AD + BE + CF \geq \frac{9R}{2}$.

Mihály Bencze

PP. 17018. Let $ABCD$ be a convex quadrilateral, and O denote the intersection point of diagonals; O_1, O_2, O_3, O_4 denote the center of circumcircles of triangles AOB, BOC, COD, DOA.

1). Determine $\frac{Area[O_1O_2O_3O_4]}{Area[ABCD]} = t$

2). Prove that $t \geq \frac{1}{2}$

3). Determine all quadrilaterals $ABCD$ such that $t = \frac{1}{2}$.

Mihály Bencze

PP. 17019. If p_i $(i = 1, 2, ..., 2k)$ are prime, $n \geq 2$, $n \in N$ and $\sum\limits_{i=1}^{2k} p_i^n = q^2$, then q is composed.

Mihály Bencze

PP. 17020. If $a_n = \begin{cases} 1 & \text{if} \quad n \text{ is Fibonacci number} \\ 0 & \text{if} \quad n \text{ is Lucas number} \\ 2 & \text{if} \quad F_n = L_n \\ 3 & \text{if} \quad n \notin \{F_k, L_k\} \end{cases}$, then $x = 0, a_1a_2...a_n... \in R\backslash Q$

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PP. 17021. Prove that $\binom{kp}{p} - k$ is divisible by p^2 for all p prime.

Mihály Bencze

PP. 17022. Denote $a_n = \begin{cases} 1 & \text{if} \quad n \text{ is prim} \\ 0 & \text{if} \quad n \text{ is composed} \\ 1 & \text{if} \quad n = 1 \end{cases}$, and $x = 0, a_1a_2...a_n...$ Prove that $\sin x, \cos x \in R\backslash Q$.

Mihály Bencze

PP. 17023. Determine all $n, k \in N$ for which $(k+1)^n - 1$ is divisible by k^n.

Mihály Bencze

PP. 17024. If $p \geq 5$ is a prime, then $(p-1)!$ is divisible by $p+1$.

Mihály Bencze

PP. 17025. If $a, b \in N$ and $b \geq 3$, then determine all prime p for which $\frac{p^a+1}{p^b-1}$ and $\frac{p^a-1}{p^b+1}$ are not integers.

Mihály Bencze

PP. 17026. Determine all $a_k \in N$ $(k = 1, 2, ..., n)$ for which $\left(\sum\limits_{k=1}^{n} a_k\right)^3$ is divisible by $\sum\limits_{k=1}^{n} a_k^2$.

Mihály Bencze

PP. 17027. Denote $1 = d_1 < d_2 < ... < d_k = n$ all positive divisors of n, and $D_i = d_1d_2...d_i + d_2d_3...d_{i+1} + ... + d_{k-i+1}d_{k-i+2}...d_k$. Determine all $n, i \in N$ for which n^i is divisible by D_i.

Mihály Bencze

PP. 17028. If a_{n+1} is the smallest prime divisor of $F_n + F_{n-1}$ for all $n \geq 2$, where F_n denote the n-th Fibonacci number, then
$x = 0, a_1 a_2 a_3 ... a_n ... \in R \backslash Q$.

Mihály Bencze

PP. 17029. Determine all $n \in N$ for which $(2n+1)^n$ have $2n+1$ decimals.

Mihály Bencze

PP. 17030. If $a_0 = 1$ and $a_{n+1} = 2a_n + 2^{a_n}$ for all $n \in N$, then determine all $n \in N$ for which a_n is a perfect k power.

Mihály Bencze

PP. 17031. Determine all $\alpha > 1$ for which $\sum_{k=1}^{n} [k\sqrt{\alpha}] > \left[\frac{n^2}{\sqrt{\alpha}}\right]$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 17032. If $x \in \left(0, \frac{\pi}{2}\right)$, then
$$\frac{\sqrt{1+\sin^2 x}}{\sin x} + \frac{\sqrt{1+\cos^2 x}}{\cos x} + \frac{\sin x + \cos x}{\sqrt{2}} + 2\left(\sqrt{\frac{\sin x}{4+\sin^2 x}} + \sqrt{\frac{\cos x}{4+\cos^2 x}}\right) \geq 2\left(1+\sqrt{2}\right).$$

Mihály Bencze

PP. 17033. In all triangle ABC holds
1). $s^2 + 5r^2 \geq 16Rr$
2). $\prod (m_a + m_b) + \prod (-m_a + m_b + m_c) \geq 9 m_a m_b m_c$.

Mihály Bencze

PP. 17034. If $x \in R$, then $\left(1 - \sqrt{\frac{1+4\sin^2 x}{5}}\right)^2 + \left(1 - \sqrt{\frac{1+4\cos^2 x}{5}}\right)^2 +$
$$25 \ln \left(\frac{e\left(16\sin^2 x + 10\sin x + 25\right)}{25}\right)^{\frac{1}{16\sin^2 x + 10\sin x + 25}} +$$
$$+25 \ln \left(\frac{e\left(16\cos^2 x + 10\cos x + 25\right)}{25}\right)^{\frac{1}{16\cos^2 x + 10\cos x + 25}} \geq 2.$$

Mihály Bencze

PP. 17035. Determine all $x, y, z \in C$ and $n \in N$ such that $(x-y)^n + (y-z)^n + (z-x)^n = n(x-y)(y-z)(z-x)$.

Mihály Bencze

PP. 17036. If $x \in \left(0, \frac{\pi}{2n}\right)$, then
$$\sum_{k=1}^{n} \frac{1}{\sqrt[3]{(\sin kx + \cos kx)(1 - \sin kx \cos kx)}} \geq \frac{\sin \frac{nx}{2}\left(\sin \frac{(n+1)x}{2} + \cos \frac{(n+1)x}{2}\right)}{\sqrt[3]{2} \sin \frac{x}{2}}.$$

Mihály Bencze

PP. 17037. In all triangle ABC holds $\sum \frac{1}{1 - \sqrt{\frac{r}{s} tg \frac{A}{2}}} \leq \frac{9}{2}$.

Mihály Bencze

PP. 17038. In all triangle ABC holds:

1). $\frac{s^2 - r^2 - 4Rr}{s^2 + r^2 + 4Rr} + \frac{16Rr}{2s(s^2 + r^2 + 2Rr)} \geq 1$

2). $\left(\frac{4R+r}{s}\right)^2 + \frac{2r}{R} \geq 2$

3). $\frac{8R^2 + r^2 - s^2}{s^2 + r^2 - 8Rr} + \frac{8Rr^2}{(2R-r)(s^2 + r^2 - 8Rr) - 2Rr^2} \geq 1$

4). $\frac{(4R+r)^2 - s^2}{(4R+r)^2 + s^2} + \frac{8Rs^2}{(4R+r)^3 + s^2(2R+r)} \geq 1$

Mihály Bencze

PP. 17039. In all triangle ABC holds:

1). $\sum \frac{r_a}{r_b} \geq 9 - \frac{18s^2}{(4R+r)^2}$

2). $\sum \frac{\sin^2 \frac{A}{2}}{\sin^2 \frac{B}{2}} \geq \frac{9\left(8R^2 + r^2 - s^2\right)}{2(2R-r)^2}$

3). $\sum \frac{\cos^2 \frac{A}{2}}{\cos^2 \frac{B}{2}} \geq \frac{9\left((4R+r)^2 - s^2\right)}{2(4R+r)^2}$

Mihály Bencze

PP. 17040. In all triangle ABC holds:

1). $\sum a^3 b \geq \frac{2}{3}\left(s^4 - \left(r^2 + 4Rr\right)^2\right)$

2). $\sum r_a^3 r_b \geq \frac{s^2}{3}\left((4R+r)^2 - 2s^2\right)$

3). $\sum \sin^6 \frac{A}{2} \sin^2 \frac{B}{2} \geq \frac{\left(s^2 + r^2 - 8Rr\right)\left(8R^2 + r^2 - s^2\right)}{384R^4}$

4). $\sum \cos^6 \frac{A}{2} \cos^2 \frac{B}{2} \geq \frac{(4R+r)^4 - s^4}{384R^4}$

Mihály Bencze

PP. 17041. In all triangle ABC holds:
1). $\sum \frac{a}{b} \geq \frac{4\left(s^2-r^2-Rr\right)}{s^2+r^2+2Rr}$
2). $\sum \frac{-a+b+c}{a-b+c} \geq 2\left(\frac{s^2+r^2}{4Rr}-2\right)$

Mihály Bencze

PP. 17042. In all triangle ABC holds:
1). $\sum \frac{a^2}{b} \geq \frac{2\left(2s^2-3r^2-12Rr\right)}{s}$
2). $\sum \frac{(s-a)^2}{s-b} \geq \frac{5s^2-12r^2-48Rr}{s}$
3). $\sum \frac{r_a^2}{r_b} \geq \frac{5(4R+r)^2-7s^2}{4R+r}$
4). $\sum \frac{\sin^4 \frac{A}{2}}{\sin^2 \frac{B}{2}} \geq \frac{20R^2+2r^2-3s^2+4Rr}{2R(2R-r)}$
5). $\sum \frac{\cos^4 \frac{A}{2}}{\cos^2 \frac{B}{2}} \geq \frac{2(4R+r)^2-3s^2}{2R(4R+r)}$

Mihály Bencze

PP. 17043. In all triangle ABC holds
1). $\sum \frac{1}{a^2+bc} \leq \frac{5s^2+r^2+4Rr}{(s^2+r^2+4Rr)(s^2+r^2+2Rr)}$
2). $\sum \frac{1}{r_a^2+r_br_c} \leq \frac{(4R+r)\left((4R+r)^2+s^2\right)}{4s^4R}$

Mihály Bencze

PP. 17044. In all triangle ABC holds:
1). $\sum \frac{1}{tg\frac{A}{2}+tg\frac{B}{2}} \geq \frac{5}{2}$
2). $\left(\sum tg^2\frac{A}{2}tg^2\frac{B}{2}\right)\prod\left(1+tg^2\frac{A}{2}tg^2\frac{B}{2}+tg^2\frac{B}{2}tg^2\frac{C}{2}\right) \leq 4$
3). $\sum tg^4\frac{A}{2}tg^4\frac{B}{2}\left(1-3tg\frac{A}{2}tg\frac{B}{2}\right) \leq \frac{1}{12}$

Mihály Bencze

PP. 17045. In all triangle ABC holds:
1). $\sum \frac{r_a}{r_b+r_c} \leq \frac{(4R+r)^2}{6s^2r}$
2). $\sum \frac{\sin^2 \frac{A}{2}}{\sin^2 \frac{B}{2}+\sin^2 \frac{C}{2}} \leq \frac{(2R-r)\left(s^2+r^2-8Rr\right)}{12Rr^2}$
3). $\sum \frac{\cos^2 \frac{A}{2}}{\cos^2 \frac{B}{2}+\cos^2 \frac{C}{2}} \leq \frac{(4R+r)\left((4R+r)^2+s^2\right)}{12Rs^2}$

Mihály Bencze

PP. 17046. In all triangle ABC holds:

1). $\sum \frac{\sin^2 \frac{A}{2}}{\sin \frac{B}{2}} \geq \sqrt[4]{\frac{27(8R^2+r^2-s^2)}{8R^2}}$

2). $\sum \frac{\cos^2 \frac{A}{2}}{\cos \frac{B}{2}} \geq \sqrt[4]{\frac{27\left((4R+r)^2-s^2\right)}{8R^2}}$

Mihály Bencze

PP. 17047. If $x \in R$ then in all triangle ABC holds

$$\left(\sum \left(tg\frac{A}{2}\right)^{\sin^2 x}\right)\left(\sum \left(tg\frac{A}{2}\right)^{\cos^2 x}\right)\left(\sum \left(ctg\frac{A}{2}\right)^{\sin^2 x}\right)\left(\sum \left(ctg\frac{A}{2}\right)^{\cos^2 x}\right) \geq$$
$$\geq 8\left(\frac{4R+r}{s}\right)^2.$$

Mihály Bencze

PP. 17048. In all triangle ABC holds $\sum \frac{ctg\frac{A}{2}}{\sqrt{ctg\frac{A}{2}+ctg\frac{C}{2}}} \leq \frac{5}{4}\sqrt{\frac{s}{r}}$.

Mihály Bencze

PP. 17049. In all triangle ABC holds:

1). $\frac{s^2+r^2}{8Rr} \geq \frac{9}{2} + \frac{8\left(s^2-r^2-4Rr\right)}{s^2+r^2+4Rr}$

2). $\frac{R}{r} + 1 \geq \left(\frac{4R+r}{s}\right)^2$

3). $\frac{(2R-r)\left(s^2+r^2-8Rr\right)}{2Rr^2} \geq 5 + \frac{8\left(8R^2+r^2-s^2\right)}{s^2+r^2-8Rr}$

4). $\frac{(4R+r)\left((4R+r)^2+s^2\right)}{2Rs^2} \geq 5 + \frac{8\left((4R+r)^2-s^2\right)}{(4R+r)^2+s^2}$.

Mihály Bencze

PP. 17050. In all triangle ABC holds

$$4s \leq 4r\max\left\{\left|ctg\frac{A}{2} - ctg\frac{B}{2}\right|; \left|ctg\frac{B}{2} - ctg\frac{C}{2}\right|; \left|ctg\frac{C}{2} - ctg\frac{A}{2}\right|\right\} + \frac{9r(4R+r)}{s} + 27r.$$

Mihály Bencze

PP. 17051. In all triangle ABC holds $tg\frac{B}{2}\left(ctg\frac{A}{2} + ctg\frac{B}{2}\right) +$
$tg\frac{C}{2}\left(\frac{2s}{r} + ctg\frac{B}{2}\right) + 2ctg\frac{B}{2}\left(\frac{2s}{r} + ctg\frac{C}{2}\right) + 2ctg\frac{C}{2}\left(ctg\frac{C}{2} + ctg\frac{A}{2}\right) \leq \frac{4s^2}{r^2}$.

Mihály Bencze

PP. 17052. If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\prod\limits_{k=1}^{n} a_k = 1$, then
$\sum\limits_{k=1}^{n} \frac{1}{a_k} + \frac{7n-4}{2\sum\limits_{k=1}^{n} a_k} \geq \frac{2n^2+7n-4}{2n}$.

Mihály Bencze

PP. 17053. If $a, b, c > 0$, then $\sum a^2 (a-b) \geq \frac{2}{3} \sum (a-b)(b-c)^2$.

Mihály Bencze

PP. 17054. In all triangle ABC holds:

1). $24s^2 \geq r(4R+r) + 81r^2 \left((4R+r)^2 - 2s^2\right)$
2). $\frac{32R}{3r} + \frac{180r(4R+r)}{s^2} \geq \frac{235}{3}$

Mihály Bencze

PP. 17055. In all triangle ABC holds:

1). $\sum \sqrt{\frac{s - rctg\frac{A}{2}}{s + rctg\frac{A}{2}}} \geq 1 + \frac{2\sqrt{3}}{3}$
2). $\sum \frac{ctg\frac{A}{2}}{\sqrt{ctg\frac{A}{2} + 2ctg\frac{B}{2}}} \leq \sqrt{\frac{3s}{2r}}$

Mihály Bencze

PP. 17056. In all triangle ABC holds $\prod \frac{1}{\sqrt{rctg\frac{A}{2} - s}} \geq \frac{\left(\sqrt{3}-1\right)^2}{s\sqrt{s}}$.

Mihály Bencze

PP. 17057. In all triangle ABC holds

$\sum \frac{1}{\sqrt{s} - tg\frac{A}{2}\sqrt{rctg\frac{B}{2}}} + \sum \frac{1}{\sqrt{s} - \sqrt{3rctg\frac{A}{2}}} \leq \frac{8}{\sqrt{s}}$.

Mihály Bencze

PP. 17058. In all triangle ABC holds:
1). $\frac{4R}{r} + \frac{13s}{\sqrt{10s^2 - 2r(4R+r)}} \geq \frac{55}{6}$
2). $\sum \sqrt{ctg\frac{A}{2}} \geq \left(4 - 2\sqrt{3} + \frac{4r(3R+r)}{s^2}\right) \sqrt{\frac{s}{r}}$

3). $\frac{4R}{r}+2\geq\frac{63s^2}{2(s^2+47r^2)}$

4). $\sum\sqrt{\frac{ctg\frac{A}{2}}{s+6rctg\frac{A}{2}}}\geq\sqrt{\frac{3}{r}}$

5). $\sum\sqrt{3+\sqrt{3}tg\frac{A}{2}}\geq 6$

Mihály Bencze

PP. 17059. In all triangle ABC holds
$\sum\left(stg\frac{A}{2}+rctg^2\frac{A}{2}\right)\left(stg\frac{B}{2}+rctg^2\frac{B}{2}\right)\leq\frac{3s^4}{16r^2}$.

Mihály Bencze

PP. 17060. In all triangle ABC holds
$\sum\left(a^2+bc\right)\left(b^2+ca\right)=3s^4-8Rs^2r-r^2\left((4R+r)^2-2s^2\right)$.

Mihály Bencze

PP. 17061. If $a,b,c,d>0$ and $\sum a=2$, then
$\sum a^4\geq 4abcd+\frac{1}{2}\sum(a-b)(c-d)$.

Mihály Bencze

PP. 17062. If $a,b,c>0$ and $\sum a^2\leq 3$ and $\sum a^2b^2\leq 3$, then
$\sum\frac{1}{(1+a)^2}+\frac{1}{1+\sum a}\leq 1$.

Mihály Bencze

PP. 17063. If $a,b,c>0$ and
$\left(\sum a\right)^3+\left(\sum ab\right)^2\geq 2\left(\sum a\right)\left(\sum ab\right)+2abc\sum a+3\sum a+3$, then
$\sum\frac{1}{(1+a)^2}+\frac{1}{1+\sum a}\geq 1$.

Mihály Bencze

PP. 17064. In all triangle ABC holds $2s^3+5r^2+20Rr+1\geq 14sr^2$. When holds the equality?

Mihály Bencze

PP. 17065. If $a,b,c>0$ and $\lambda\in R$, then
$\sum a^4+\left(3\lambda^2-1\right)\sum a^2b^2+3\lambda(1-\lambda)abc\sum a\geq 3\lambda\sum a^2b$.

Mihály Bencze

PP. 17066. In all triangle ABC holds $\prod\left(s^3+r^3ctg^3\frac{A}{2}\right)\geq s^6\left(s-r\right)^3.$

Mihály Bencze

PP. 17067. If $a,b,c,d>0$, then
$2\prod\left(1+a^2\right)\geq(|a+b+c+d-abc-bcd-cda-dab|+$
$+|1+abcd-ab-bc-cd-da-ac-bd|)^2.$

Mihály Bencze

PP. 17068. If $a,b,c,d\in R$, then
$2a\left(a+b\right)^5+2b\left(b+c\right)^5+2c\left(c+d\right)^5+2d\left(d+a\right)^5+\left(a+c\right)^6+\left(b+d\right)^6\geq 0.$

Mihály Bencze

PP. 17069. In all triangle ABC holds: $\prod\left(1-\sqrt{\frac{2r}{3s}ctg\frac{A}{2}}\right)\geq\frac{\sqrt{6}\left(\sqrt{3}+1\right)r}{9s}.$

Mihály Bencze

PP. 17070. In all acute triangle ABC holds:
$\sum\frac{2a^4-b^2c^2}{b^4-b^2c^2+c^4}\leq 3+2\sum\frac{ab\left(a^2+b^2\right)\left(a^2-b^2\right)^2}{(a^4-a^2c^2+c^4)(b^4-b^2c^2+c^4)}.$

Mihály Bencze

PP. 17071. In all triangle ABC holds $1+\frac{r^4}{s^4}\sum ctg^4\frac{A}{2}\geq\frac{4r^2}{s^2}+$
$+\frac{2r}{s}\max\left\{\left(ctg\frac{A}{2}-ctg\frac{B}{2}\right)\left(tg\frac{A}{2}tg\frac{B}{2}-1\right);\left(ctg\frac{B}{2}-ctg\frac{C}{2}\right)\left(tg\frac{B}{2}tg\frac{C}{2}-1\right);\right.$
$\left.\left(ctg\frac{C}{2}-ctg\frac{A}{2}\right)\left(tg\frac{C}{2}tg\frac{A}{2}-1\right)\right\}.$

Mihály Bencze

PP. 17072. If $a,b,c\in R$, then
$2\left(1+a^3b^3c^3\right)+\sqrt{2\left(1+a^6\right)\left(1+b^6\right)\left(1+c^6\right)}\geq\left(ab+bc+ca-abc\right)^3.$

Mihály Bencze

PP. 17073. If $a,b,c>0$, then $\left(\sum a\right)\left(\sum a^2-\sum ab\right)\geq 9\prod\left(a-\sqrt{bc}\right).$

Mihály Bencze

PP. 17074. If $a,b,c>0$, then
$\left(a^2+ab+b^2\right)\left(b^2+bc+c^2\right)\left(c^2+ca+a^2\right)\geq\frac{3}{4}\left(\sum ab\left(a+b\right)\right)^2.$

Mihály Bencze

PP. 17075. In all triangle ABC holds
$\sum\sqrt[3]{\frac{1}{2}\left(ctg^3\frac{A}{2}+ctg^3\frac{B}{2}\right)}\leq\frac{3\left(s^2-2r^2-8Rr\right)}{sr}$.

Mihály Bencze

PP. 17076. If $a,b,c,d>0$, then
$\frac{a+d}{\sqrt{8b^2+c^2}}+\frac{b+c}{\sqrt{8d^2+b^2}}+\frac{c+d}{\sqrt{8a^2+b^2}}+\frac{a+b}{\sqrt{8c^2+d^2}}+\frac{a}{\sqrt{8b^2+d^2}}+\frac{c}{\sqrt{8d^2+a^2}}+\frac{d}{\sqrt{8a^2+c^2}}\geq 4.$

Mihály Bencze

PP. 17077. Determine all prime p and q such that the sequence
$x_n=p^n-q+1$ contain m consecutive term where are composite.

Mihály Bencze

PP. 17078. Determine all $a_k\in N$ $(k=1,2,...,n)$ such that
$(a_1,a_2,...,a_n)=\left(\frac{a_1+a_2+...+a_{n-1}}{n-1},\frac{a_2+a_3+...+a_n}{n-1},...,\frac{a_n+a_1+...+a_{n-2}}{n-1}\right)$.

Mihály Bencze

PP. 17079. Determine all prime p and q for which $3^p=2010+q^3$.

Mihály Bencze

PP. 17080. Determine all $k\in N$ for which the sequence
$\left\{n^k+(n+1)^k\,|n\in N\right\}$ contain an infinitely many composite numbers.

Mihály Bencze

PP. 17081. Determine all prime p and q such that p^n+q is composite for all $n\in N$ but is not perfect square.

Mihály Bencze

PP. 17082. Determine all $a_k\in N$ $(k=1,2,...,n)$ such that
$(a_1,a_2)=(a_2,a_3)+1=(a_3,a_4)+2=...=(a_n,a_1)+n-1$.

Mihály Bencze

PP. 17083. Determine all $p,n\in N$ such that $(n!)^p$ is divisible by $\sum\limits_{k=1}^{n}k^p$.

Mihály Bencze

PP. 17084. Determine all $n, a, b, c \in N$ for which $a^{n+1} + b^{n+1} + c^{n+1}$ is divisible by $a^n + b^n + c^n$.

Mihály Bencze

PP. 17085. Determine all $a_k \in N$ $(k = 1, 2, ..., n)$ such that
$[a_1, a_2, ..., a_n] \prod_{1 \le i < j \le n} (a_i, a_j) = (a_1, a_2, ..., a_n) \prod_{k=1}^{n} a_k.$

Mihály Bencze

PP. 17086. Determine all $m, n \in N$ such that $(5^m - 1, 5^n + 1) = 2.$

Mihály Bencze

PP. 17087. Denote p_n the $n-$ th prime, if $a_n \in N$ $(n = 1, 2, ...)$ and $a_n \ge p_n$, then $\sum_{k=1}^{n} \frac{1}{(a_n)!}$ and $\sum_{k=1}^{\infty} \frac{(-1)^n}{(a_n)!}$ are irrational numbers.

Mihály Bencze

PP. 17088. If $s(n)$ denote the sum of decimals of number n, then prove that exist infinitely many $y, x_k \in N$ $(k = 1, 2, ..., n)$ such that $\sum_{k=1}^{n} s\left(x_k^2\right) = s\left(y^2\right).$

Mihály Bencze

PP. 17089. Prove that the set $\left\{\left[n\sqrt{p}\right], n \in N, p \ge 2 \text{ prime}\right\}$ contain infinitely many perfect squares, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 17090. Determine all prime p and q for which $\sqrt{n+p} + \sqrt[3]{n+q}$ are irrational numbers for all $n \in N$.

Mihály Bencze

PP. 17091. Solve in N the equation $x^2 - \left(y^2 - 1\right) z^2 = 1.$

Mihály Bencze

PP. 17092. Solve in N the equation
$\left(x^2-1\right)\left(y^2-1\right)\left(z^2-1\right)=\left(\left(\frac{x+y+z}{3}\right)^2-1\right)^2.$

Mihály Bencze

PP. 17093. Denote $x\left(n\right)$ the number of decimals ≥ 5 in decimal representations of n. Compute $\sum\limits_{n=0}^{\infty}\frac{x(3^n)}{3^n}$.

Mihály Bencze

PP. 17094. Prove that $\sum\limits_{k=1}^{n}d\left(k\right)\sigma\left(k\right)\geq n^2+3n-3.$

Mihály Bencze

PP. 17095. Prove that
$\sum\limits_{k=1}^{n}d^2\left(k\right)\varphi^2\left(k\right)\geq\frac{n(n+1)(2n+1)}{6}\geq -1+\sum\limits_{k=1}^{n}\sigma\left(k\right)\varphi\left(k\right).$

Mihály Bencze

PP. 17096. Solve in N the equation $2011\cdot 4^n=x^2+y^2+z^2+t^2$.

Mihály Bencze

PP. 17097. In all triangle ABC holds:
1). $\sum m_ah_bh_c\leq 3s^2r$
2). $\sum am_a^2h_bh_c\leq 3s^3Rr$

Mihály Bencze

PP. 17098. Let $ABCD$ be a convex quadrilateral. A circle with trough in A intersect second time AB, AC, AD in M,N,K. Determine all convex quadrilaterals for which $AM\cdot AB+AK\cdot AD=AN\cdot AC$.

Mihály Bencze

PP. 17099. If exist $x,y,z\in R$ such that $xr_a+yr_b+zr_c=R$ in a triangle ABC, then determine the function $f:R\rightarrow R$ for which
$f\left(x\right)\cos A+f\left(y\right)\cos B+f\left(z\right)\cos C=0.$

Mihály Bencze

PP. 17100. In all triangle ABC holds
$\frac{s}{R}+\left(1+\sqrt{2}\right)\sum\cos\frac{A}{2}\leq\left(2+\sqrt{2}\right)\sum\cos\frac{A-B}{2}\cos\frac{C}{2}.$

Mihály Bencze

PP. 17101. Let $A_1B_1C_1D_1A_2B_2C_2D_2$ be a rectangle parallelipipedon. A sphere trough A_1 intersect A_1C_1, A_1C_2, A_1D_2 in M,N,K. Prove that
$A_1M\cdot A_1C_1+A_1K\cdot A_1D_2=A_1N\cdot A_1C_2.$

Mihály Bencze

PP. 17102. Determine one function $f:(0,+\infty)\to(0,+\infty)$ such that
$\left(\underbrace{f\circ...\circ f}_{k-\text{ time}}\right)(x)=\frac{ax+b}{bx+a}$ where $a,b>0$ and $k\in N$ are given.

Mihály Bencze

PP. 17103. If $x_k>0$ $(k=1,2,...,n)$, then
$\sum\limits_{cyclic}\frac{\sqrt{x_1^2-x_1x_2+x_2^2}+\sqrt{x_1^2+x_1x_2+x_2^2}}{x_1x_2}\geq\sqrt{3}\sum\limits_{cyclic}\left(\frac{1}{\sqrt{x_1^2-x_1x_2+x_2^2}}+\frac{1}{\sqrt{x_1^2+x_1x_2+x_2^2}}\right).$

Mihály Bencze

PP. 17104. If $x\in\left[0,\frac{\pi}{2}\right]$, then
$\sin x+\cos x+3\sqrt{2}\leq2\left(\sqrt{1+2\sin^2x}+\sqrt{1+2\cos^2x}\right).$

Mihály Bencze

PP. 17105. If $x,y,z>0$, then $\frac{ch^2x}{shy+shz}+\frac{ch^2y}{shz+shx}+\frac{ch^2}{shx+shy}\geq3.$

Mihály Bencze

PP. 17106. If $x_k>0$ $(k=1,2,...,n)$, then determine the best constant
$c\geq\frac{9}{8}$ such that $\sum\limits_{k=1}^{n}x_k^3\geq c\sum\limits_{cyclic}x_1x_2\left|x_1-x_2\right|.$

Mihály Bencze

PP. 17107. If $x\in R$, then
$\sqrt{3}\leq2\left|\cos\frac{x}{2}\right|+\sqrt{4-2\sin x-4\cos x+2\left(\sin2x+\cos2x\right)}\leq\frac{13}{4}.$

Mihály Bencze

PP. 17108. If $a, b \geq 0$, then determine all $c \in R$ such that
$\int\limits_0^x (ct - x)(asht + bcht)\,dt \geq 0$ for all $x \in R$.

Mihály Bencze

PP. 17109. In all triangle ABC holds
$12\left(s^2 - r^2 - 4Rr\right)^2 \geq \left(s^2 + r^2 + 2Rr\right)\left(11s^2 - 15r^2 - 60Rr\right).$

Mihály Bencze

PP. 17110. If $a_k > 1$ $(k = 1, 2, ..., n)$, then
$\sum\limits_{cyclic} \log_{a_1}\left(a_1 + \sqrt[n]{a_2 a_3 ... a_n}\right) \geq \frac{2n-1}{2} + \frac{n \ln 2}{\ln\left(\frac{1}{n}\sum\limits_{k=1}^{n} a_k\right)}.$

Mihály Bencze

PP. 17111. If $a, b, c > 0$, then
$\left(\sum a^2 + \sum ab\right)^3 \geq 3\left(\sum a\right)\left(\sum (a+b)\left(a^2b^2 + c^4\right) + 4abc\sum a\right).$

Mihály Bencze

PP. 17112. Let ABC be a triangle, and $M \in Int(ABC)$. Denote d_a, d_b, d_c the distances from M to the sides BC, CA, AB. Determine all points M such that $r \leq \left(\frac{\sqrt{d_a}+\sqrt{d_b}+\sqrt{d_c}}{3}\right)^2 \leq \frac{R}{2}$.

Mihály Bencze

PP. 17113. If $a, b, c > 1$ then $\sum \frac{\left(a^2+bc\right)^{n+1} - (b+c)^{n+1}}{(a^2+bc-b-c)(b+c)^n} \geq \frac{(a+b+c)^{n+1} - 3^{n+1}}{3^{n-1}(a+b+c-3)}$ for all $n \in N$.

Mihály Bencze

PP. 17114. If $x_k, y_k, z_k \geq 0$ $(k = 1, 2, 3, 4)$, then
$\left(\sum\limits_{k=1}^{4} x_k\right)^2 + \left(\sum\limits_{k=1}^{4} y_k\right)^2 + \left(\sum\limits_{k=1}^{4} z_k\right)^2 \geq 4\left(x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1\right) +$
$+4\left(y_1y_2 + y_2y_3 + y_3y_4 + y_4y_1\right) + 4\left(z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1\right) +$
$+\frac{1}{3}\left(|x_1 - x_2 + x_3 - x_4| + |y_1 - y_2 + y_3 - y_4| + |z_1 - z_2 + z_3 - z_4|\right)^2.$

Mihály Bencze

PP. 17115. If $x, y, z > 0$, then
$\sum \left(54x^2y + 3(x+y+z)^3\right)\left(54y^2z + 3(x+y+z)^3\right) \leq 75(x+y+z)^6$.

Mihály Bencze

PP. 17116. If $x_k \in R$ $(k = 1, 2, ..., n)$, then
$2\sum\limits_{k=1}^{n} |\cos x_k| + \sqrt{n + 2\sum\limits_{1\leq i<j\leq n} \cos 2(x_i - x_j)} \geq n$

Mihály Bencze

PP. 17117. If $z_k \in C$ $(k = 1, 2, ..., n)$, then $\sum\limits_{k=1}^{n} |1 + z_k| + \left|\sum\limits_{k=1}^{n} z_k\right| \geq n$.

Mihály Bencze

PP. 17118. The distinct points A, B, C, D, E lie in this order on curve $a_1x^2 + a_2xy + a_3y^2 + a_4x + a_5y + a_6 = 0$ and satisfy $AC = BD = CE = r$. Determine all $a_k \in R$ $(k = 1, 2, 3, 4, 5, 6)$ such that the orthocentres of triangles ACD, BCD, BCE are the vertices of a right-angled triangle.

Mihály Bencze

PP. 17119. If $x, y, z \in R$, then
1). $2(|\cos x| + |\cos y| + |\cos z|) + |\cos(x+y)| + |\cos(y+z)| +$
$+ |\cos(z+x)| \geq 3$.
2). $|\cos x| + |\cos y| + |\cos z| + |\cos(x+y)| + |\cos(y+z)| + |\cos(z+x)| +$
$+3|\cos(x+y+z)| \geq 3$.

Mihály Bencze

PP. 17120. Find all functions $f : R \to R$ such that
$f\left(\sum\limits_{k=1}^{n} x_k^n\right) = \sum\limits_{k=1}^{n} x_1x_2...x_{k-1}f(x_k)x_{k+1}...x_n$ for all $x_k \in R$ $(k = 1, 2, ..., n)$.

Mihály Bencze

PP. 17121. Determine all $a, b \in N$ such that exist infinitely many positive integers m and n such that $\frac{m+a}{n} + \frac{n+b}{m}$ is a positive integer.

Mihály Bencze

PP. 17122. If $x, y, a, b, c > 0$, then $x + \frac{y}{ab+bc+ca} \geq (x+y)\left(\frac{3}{(a+b+c)^2}\right)^{\frac{y}{x+y}}$.

Mihály Bencze

PP. 17123. Find all primes p, q, r such that $p^q + q^r + r^p + 1$ is divisible by pqr.

Mihály Bencze

PP. 17124. If $a, b, c > 0$, then $\left(a^2+b^2\right)^2 + \left(b^2+c^2\right)^2 + \left(c^2+a^2\right)^2 \geq$
$\geq 3(a+b+c)(a+b-c)(b+c-a)(c+a-b)$.

Mihály Bencze

PP. 17125. Determine all $y, x_k \in N$ $(k = 1, 2, ..., n)$ such that
$2011 + \sum\limits_{k=1}^{n} 4^{x_k} = y^2$.

Mihály Bencze

PP. 17126. Triangle ABC is acute with circumellipse (E) and his centre O. The ellipse (E_1) has centre O_1, is tangent to O at A and to the side BC at D, and intersects the lines AB and AC again at M and N, respectively. The lines OO_1 and MO_1 intersect (E_1) again at A_1 and K, respectively. The lines BO and A_1K intersect at L. Prove that $DN^2 = AN \cdot LK$.

Mihály Bencze

PP. 17127. Determine all prime p, q and all $r, a \in N^*$ such that
$a^p - 1 = (a-1)q^r$.

Mihály Bencze

PP. 17128. If $x_1 \geq 1$, $x_1 \in Q$, and $x_{n+1} = x_n^a + \frac{1}{\left[x_n^b\right]}$ for all $n \geq 1$, where $[\cdot]$ denote the integer part. Determine all $a, b \in R$ for which this sequence contain one integer.

Mihály Bencze

PP. 17129. If $n \in N^*$, then $\sum\limits_{k=1}^{n} \frac{1}{k^2} \geq \frac{n(3n+5)}{2(n+1)^2}$.

Mihály Bencze

PP. 17130. Determine all positive integers: $x_k \in N^*$ $(k = 1, 2, ..., n)$ such that

1). $\sum\limits_{k=1}^{n} \log_a x_k = \log_a \left(\sum\limits_{k=1}^{n} x_k \right)$

2). $\sum\limits_{k=1}^{n} (-1)^{k-1} \log_a x_k = \log_a \left(\sum\limits_{k=1}^{n} (-1)^{k-1} x_k \right)$

3). $\sum\limits_{k=1}^{n} (\pm \log_a x_k) = \log_a \left(\sum\limits_{k=1}^{n} (\pm x_k) \right)$, when $a \in (0, 1) \cup (1, +\infty)$.

Mihály Bencze

PP. 17131. Determine all triangles ABC such that $\sum \frac{\sqrt{bc \cos A}}{b+c-a} \leq \frac{3\sqrt{2}}{2}$.

Mihály Bencze

PP. 17132. Let ABC be a triangle. Denote r_m and R_m the inradii and circumradii of the triangle formed by m_a, m_b, m_c. Prove that
$\sum \frac{1}{5a^2-b^2-c^2} \geq \frac{1}{8r_m R_m}$.

Mihály Bencze

PP. 17133. If $x_k > 0$ $(k = 1, 2, ..., n)$ then
$\frac{1}{16} \left(n + \sum\limits_{k=1}^{n} x_k^2 \right) \geq \sum\limits_{k=1}^{n} \left(\frac{x_k}{\sqrt{x_k^2+1+\sqrt{2}x_k}} \right)^2$.

Mihály Bencze

PP. 17134. Solve the following equations:

1). $\sum\limits_{k=1}^{n} \left[\frac{k}{x} \right] = n$

2). $\sum\limits_{k=1}^{n} \left[\frac{k^2}{x} \right] = n$

3). $\sum\limits_{k=1}^{n} \left[\frac{k}{x} \right] = \frac{n(n+1)}{2}$

4). $\sum\limits_{k=1}^{n} \left[\frac{k^2}{x} \right] = \frac{n(n+1)(2n+1)}{6}$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 17135. Determine all $a, b, c \in N^*$ and $x, y, z, t, u, v \in Z$ for which $a^{xn+y} + b^{zn+t} + c^{un+v}$ is divisible by $a^y + b^t + c^v$ for all $n \in N$.

Mihály Bencze

PP. 17136. If $a > 0$ and $x \in (-a, a)$ such that $3 \geq \frac{2}{a^2-x^2} + \frac{1}{a^2}$, then $a^2\left(a^2 - x^2\right) \geq 1$.

Mihály Bencze

PP. 17137. If $x_n = \frac{L_n}{L_{n+1}}$ when L_n denote the n^{th} Lucas number and $n \geq 1$, then determine all $a, b \in R$ such that $x_{n+2} = ax_{n+1} + bx_n$ for all $n \in N^*$.

Mihály Bencze

PP. 17138. If $x_{n+1} = \frac{ax_n+b}{cx_n+d}$, where $x_1 = 1$ and $a, b, c, d \in Z$, then express x_n in terms of Fibonacci or Lucas numbers.

Mihály Bencze

PP. 17139. Let $T_n = \frac{n(n+1)}{2}$ denote the n^{th} triangular number. Find a formula for T_{kn} in terms of T_n when $k \in N$, $k \geq 2$.

Mihály Bencze

PP. 17140. Let $A_n = \begin{pmatrix} F_n & L_n & L_n \\ L_n & F_n & L_n \\ L_n & L_n & F_n \end{pmatrix}$ where L_n and F_n are the n^{th} Lucas and Fibonacci numbers. Find a formula for A_{kn} in terms of $A_n, A_{n+1}, ..., A_{n+k-1}$.

Mihály Bencze

PP. 17141. An ellipse is inscribed in triangle ABC such that is tangent to his sides and the one axes of ellipse is parallel to BC. Let DE be a chord of the ellipse parallel to side BC. If $BD = CE$, prove that $AB = AC$.

Mihály Bencze

PP. 17142. Let M be a point inside n-gon $A_1A_2...A_n$. Let $B_1, B_2, ..., B_n$ be the centroids of triangles. $A_1MA_2, A_2MA_3, ..., A_nMA_1$. Determine all $n \in N$ for which the segments $A_1B_1, A_2B_2, ..., A_nB_n$ are concurrent.

Mihály Bencze

PP. 17143. If $S_n = (x-y)^n + (y-z)^n + (z-x)^n$. Determine all $a, b \in N^*$ such that S_{an+b} is divisible by $\frac{S_b}{b}$ for all $n \in N$.

Mihály Bencze

PP. 17144. An ellipse intersects each side of a regular n-gon $A_1A_2...A_n$ in two points. The ellipse cuts side A_iA_{i+1} in points B_i and C_i with B_i lying between A_i and A_{i+1} and C_i lying between B_i and A_{i+1}. Prove that $\sum\limits_{i=1}^{n} A_iB_i = \sum\limits_{i=1}^{n} C_iA_{i+1}$.

Mihály Bencze

PP. 17145. Prove that each root of the equation $\sum\limits_{k=0}^{n} L_{n+k}x^{n-k} = 0$ has absolute value near $\frac{1+\sqrt{5}}{2}$, where L_n denote the n^{th} Lucas number.

Mihály Bencze

PP. 17146. Prove that $3^{2n+3} - 128n(n+1)(2n+1) + 228n^2 + 264n - 27$ is divisible by 12288.

Mihály Bencze

PP. 17147. Determine all $a, b, c \in N$ such that $L_n \equiv F_{an+b} \pmod{c}$ for all $n \in N$, where L_n and F_n denote the n^{th} Lucas and Fibonacci numbers.

Mihály Bencze

PP. 17148. Determine all $a, b, c \in N$ such that $F_n \equiv L_{an+b} \pmod{c}$ for all $n \in N$, where L_n and F_n denote the n^{th} Lucas and Fibonacci numbers.

Mihály Bencze

PP. 17149. Over C the polynomial $x^2 + y^2$ factors as $(x+iy)(x-iy)$. Determine all $n \in N$ for which $\sum\limits_{k=1}^{n} x_k^2$ factor over C.

Mihály Bencze

PP. 17150. An arboles is formed by erecting semiellipses on segments AC, CB and AB, $D \in \overset{\frown}{AB}$. An ellipse with center P, is drawn touching the semiellipse on BC at E and touching the semiellipse on AB and tangent to CD, $\{F\} = EP \cap CD$. Prove that $EF = AC$.

Mihály Bencze

PP. 17151. Let $ABCD$ be a convex quadrilateral and let E be any point on side AD. Let r_1, r_2, r_3 represent the inradii of triangles ABE, BEC, CED. Determine all convex quadrilaterals $ABCD$ such that $r_1 + r_3 \geq r_2$.

Mihály Bencze

PP. 17152. Prove that:
1). $\prod\limits_{k=1}^{n} \binom{n}{k}^{\frac{1}{k}} \leq \left(\frac{1}{1+\ln n} \sum\limits_{k=1}^{n} \frac{2^k-1}{k} \right)^{\ln(n+1)}$
2). $\left(\sum\limits_{k=1}^{n} \frac{2^k-1}{k} \right)^2 \leq \left(\binom{2n}{n} - 1 \right) \left(2 - \frac{1}{n} \right)$

Mihály Bencze

PP. 17153. Determine all $a_k \in C$ $(k = 1, 2, ..., n)$ for which $\sum\limits_{k=1}^{n} a_k, \sum\limits_{1 \leq i < j \leq n} a_i a_j, ..., \sum\limits_{1 \leq i_1 < ... < i_k \leq n} a_{i_1} a_{i_2} ... a_{i_k}, ..., \prod\limits_{k=1}^{n} a_k$ forms an arithmetical progression. Same question for a geometrical progression.

Mihály Bencze

PP. 17154. Prove that $15^{n+1} - 2^{3n+2} + 14n + 9$ is divisible by 1372.

Mihály Bencze

PP. 17155. Let P be the center of a regular $n-$gon erected outwardly on side A_1A_2 of an regular $k-$gon. Also let Q be the center of regular $m-$gon erected outwardly on side A_1A_k. If R is the midpoint of A_2A_k, then determine all $n, k, m \in N^*$ such that PRQ is a right angle.

Mihály Bencze

PP. 17156. Let ABC be an equilateral triangle with center O. Prove that, if M is a variable point on a fixed ellipse with center O, then the triangle whose sides have lengths MA, MB, MC has a constant area.

Mihály Bencze

PP. 17157. Let $F_n = \frac{a_i}{b_i}$ $(i = 1, 2, ..., m)$, be the Farey sequence of order n, that is, the ascending sequence of irreductible fractions between 0 and 1 whose denominators do not exceed n.

For example $F_5 = \left(\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, 1\right)$ with $m = 11$. If $A_0\left(0, 0\right)$, $A_i\left(a_i, b_i\right)\ (i = 1, 2, ..., m)$, then determine $Area\left[B_0 B_1 ... B_m\right]$ where B_i is the midpoint of $A_i A_{i+1}$.

Mihály Bencze

PP. 17158. Prove that:
1). $4^{n+3} + 9^{n+2} - 25 \equiv 0 \,(\text{mod } 120)$
2). $2^{n+3} + 3^{2n+3} - 19 \equiv 0 \,(\text{mod } 56)$
3). $2^{8n+12} + 85 \cdot 3^{n+3} - 2295 \equiv 0 \,(\text{mod } 2816)$
4). $2^{4n+7} + 15 \cdot 3^{n+2} - 53 \equiv 0 \,(\text{mod } 390)$
5). $2^{6n+9} + 3^{4n+6} - 17 \equiv 0 \,(\text{mod } 17680)$
6). $2^{3n+8} + 7 \cdot 3^{3n+4} - 53 \equiv 0 \,(\text{mod } 266)$
7). $2^{5n+7} + 31 \cdot 3^{n+4} - 839 \equiv 0 \,(\text{mod } 1748)$
8). $182 \cdot 4^{4n+6} + 31 \cdot 3^{6n+16} - 1147 \equiv 0 \,(\text{mod } 1241784)$ for all $n \in N$.

Mihály Bencze

PP. 17159. Solve the following system: $\begin{cases} x\left(1 + y + yz\right) = 7 \\ y\left(1 + z + zx\right) = 14 \\ z\left(1 + x + xy\right) = 12 \end{cases}$.

Mihály Bencze

PP. 17160. Find a necessary and sufficient condition on a, b, c, d so that the roots of the equation $x^4 + ax^3 + bx^2 + xc + d = 0$ are the vertices of a square in the complex plane. Generalization.

Mihály Bencze

PP. 17161. A circle Γ with center G is drawn tangent to the two legs and tangent internally to the circumcircle of the triangle ABC, touching the circumcircle in H. Find all triangle ABC and the radius of Γ such that GH is parallel to AB.

Mihály Bencze

PP. 17162. Find in terms of a, b, c, d a formula for the area of a quadrilateral whose vertices are the roots of $x^4 - ax^3 + bx^2 - cx + d = 0$ in the complex plane. Generalization.

Mihály Bencze

PP. 17163. Prove that $\left(15^n - 2^{3n+1} + 1\right)\left(9^n - 32n^2 + 24n - 1\right)$ is divisible by 50176 for all $n \in N$.

Mihály Bencze

PP. 17164. If F_n denote the n^{th} Fibonacci number, then
$\prod\limits_{k=0}^{n} F_k^{\binom{n}{k}} \leq \left(2^{-n} F_{2n}\right)^{2^n}$.

Mihály Bencze

PP. 17165. Let ABC be a triangle, $D \in (BC)$. A perpendicular to AC erected at C meets AD extended at point E. If $BAD\measuredangle = xDAC\measuredangle$ then determine all point $D \in (BC)$ such that $AE = xAB$.

Mihály Bencze

PP. 17166. Let A_n be an $n \times n$ determinant in which the entries, 1 to n^2, are put in order along the diagonals. For example, $A_4 = \begin{vmatrix} 1 & 2 & 4 & 7 \\ 3 & 5 & 8 & 11 \\ 6 & 9 & 12 & 14 \\ 10 & 13 & 15 & 16 \end{vmatrix}$.

Compute:

1). $\sum\limits_{k=1}^{n} A_k$ 2). $\sum\limits_{k=1}^{n} A_k^2$ 3). $\sum\limits_{k=1}^{n} (-1)^k A_k$ 4). $\sum\limits_{k=1}^{n} (-1)^k A_k^2$

Mihály Bencze

PP. 17167. If x_n denote the positive root of the equation $x^{-n} = \ln\left(x^\alpha + e\right)$ where $n \in N$, then determine all $\alpha \in R$ such that $\lim\limits_{n\to\infty} x_n = \alpha$.

Mihály Bencze

PP. 17168. Let ABC be a triangle, the equilateral triangle ABD is erected outwardly on side AB. Determine the angle C such that CA, CB, CD can be the sides of a triangle.

Mihály Bencze

PP. 17169. Determine all $k \in N^*$ such that if $a_1 \geq k$, $a_{n+1} = a_n^k - a_n + 1$ for all $n \geq 1$ then $\sum\limits_{i=1}^{n} \frac{1}{a_i} < k - 1$.

Mihály Bencze

PP. 17170. Determine all polygons $A_1A_2...A_n$ inscribed in circle $C(O,R)$ such that exist $i \in \{1,2,...,n\}$ for which $\sum\limits_{\substack{k=2\\k\neq i}}^{n-1} A_iA_k^{\alpha} \geq (n-2)\left(\frac{(n-1)n^{\frac{1}{n-1}}R}{n-2}\right)^{\alpha}$ for all $\alpha \geq 1$.

Mihály Bencze

PP. 17171. Determine all $a_k \in R$ $(k = 1,2,...,n)$ for which the equations: $x^2 + x\sum\limits_{k=1}^{n-1} a_k + a_n = 0, x^2 + x(a_2 + a_3 + ... + a_n) + a_1 =$ $= 0, ..., x^2 + x(a_n + a_1 + ... + a_{n-2}) + a_{n-1} = 0$ have a common root.

Mihály Bencze

PP. 17172. Determine all prime p and q and all $n \in N$ such that $p^n - q^n$ is divisible by $10n$.

Mihály Bencze

PP. 17173. If in triangle ABC we have $4s^2R = 1$, then $s^2 \leq \frac{3}{4}$.

Mihály Bencze

PP. 17174. In all triangle ABC holds $\left(\sum m_a\right)^3 \prod(-m_a + m_b + m_c) \leq 27m_a^2m_b^2m_c^2$.

Mihály Bencze

PP. 17175. If $a_k > 0$ $(k = 1,2,...,n)$, then $\sum\limits_{k=1}^{n} a_k^3 \geq \sum\limits_{cyclic} a_1a_2a_3 + \frac{3}{4}\sum\limits_{cyclic} |(a_1 - a_2)(a_2 - a_3)(a_3 - a_1)|$.

Mihály Bencze

PP. 17176. In all triangle ABC holds $\sum \frac{1}{1+3\sqrt{3}\left(tg\frac{B}{2}+tg\frac{C}{2}\right)tg^2\frac{A}{2}} \leq \frac{s\sqrt{3}}{9r}$.

Mihály Bencze

PP. 17177. Find all integers $n \in N$ such that $[\sqrt[3]{n}] + 1$ divides $n - 1$ and $[\sqrt[3]{n}] - 1$ divides $n + 1$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 17178. 1). In all triangle ABC holds $\sum \frac{1}{\cos \frac{A}{2}} \leq \frac{s^2-r^2-4Rr}{sr}$
2). Compute $\max \sum \frac{1}{\sin \frac{A}{2}}$

Mihály Bencze

PP. 17179. Denote $\lambda(n)$ the sum of digit 1 in transformation of number n in system of base two. Determine all $p \in N$ such that $\lambda(n^p) \leq \sum_{k=1}^{\lambda(n)} k^p$.

Mihály Bencze

PP. 17180. In all triangle ABC holds $18R + r \geq \sqrt{73r^2 + 72s^2}$.

Mihály Bencze

PP. 17181. Determine all $a, n \in N^*$ for which $\frac{a^{a^{n+1}}-1}{a^{a^n}-1}$ is composite number.

Mihály Bencze

PP. 17182. If $a_k > 0$ $(k = 1, 2, ..., n+1)$ and $\lambda_i > 0$ $(i = 1, 2, ..., n)$ such that $\lambda = \lambda_1 + \lambda_n = \lambda_2 + \lambda_{n-1} = \lambda_3 + \lambda_{n-2} = ...$, then
$\sum_{cyclic} \frac{a_1}{\lambda_1 a_2 + \lambda_2 a_3 + ... + \lambda_n a_{n+1}} \geq \frac{2(n+1)}{\lambda n}$.

Mihály Bencze

PP. 17183. In all triangle ABC holds:
1). $\sum \frac{(b+c)w_a^2}{s-a} = \frac{2\left(\left(s^2+r^2+4Rr\right)^2+8s^2Rr\right)}{s^2+r^2+2Rr}$
2). $\sum \frac{w_a^2+w_b^2}{c\sqrt{(s-a)(s-b)}} \geq \frac{16s}{s^2+r^2+2Rr}$

Mihály Bencze

PP. 17184. Compute
1). $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^k}{k^i}$
2). $\lim_{k\to\infty} \sum_{i=1}^{n} \frac{i^k}{k^i}$

Mihály Bencze

PP. 17185. If $a_k \geq 0$ $(k = 1, 2, ..., n)$ and $\sum\limits_{k=1}^{n} a_k = 1$, and $z_k \in C$ $(k = 1, 2, ..., n)$, then
$\frac{n^2-n+2}{2} \sum\limits_{k=1}^{n} a_k |z_k|^2 \geq \left(\left| \sum\limits_{k=1}^{n} a_k z_k \right| + \sum\limits_{1 \leq k < p \leq n} a_k a_p |z_k - z_p| \right)^2.$

Mihály Bencze

PP. 17186. In all triangle ABC holds $\sqrt{\left(\frac{b-c}{a}\right)^2 - 1} \leq \frac{s}{r}$.

Mihály Bencze

PP. 17187. If $x, y, z > 0$ and
$\left(\sum x\right)^4 = 3 + 2\sum x + 2\sum x^2 + \sum x^3 + \sum xy + \sum x^2y^2 + \sum x^2y$, then
$\sum \frac{1}{(x^2+y+1)(y^2+z+1)} \leq 1.$

Mihály Bencze

PP. 17188. If $x, y, z, t > 0$ and
$xy + xz + xt + yz + yt + zt = 2 + x + y + z + t$, then $\sum \frac{1}{x^2+y+z+1} \leq 1$.

Mihály Bencze

PP. 17189. If $x \in \left(0, \frac{\pi}{2}\right)$, then $\sum\limits_{k=0}^{n} \log_2 \left((\sin x)^{2^{-k}} + (\cos x)^{2^{-k}} \right) \leq n + \frac{1}{2^{n+1}}$.

Mihály Bencze

PP. 17190. If $x \in \left(0, \frac{\pi}{2}\right)$, then
$\frac{(1+arctgx)^{n+1}-1}{arctgx} + \frac{(1+arcctgx)^{n+1}-1}{arcctgx} \geq \frac{8}{\pi} \left(\left(1 + \frac{\pi}{4}\right)^{n+1} - 1 \right)$ for all $n \in N$.

Mihály Bencze

PP. 17191. If $b \geq a > 0$, then $\frac{\pi(b-a)}{4} + arctg\left(\frac{shb-sha}{1+shbsha}\right) \geq \frac{\ln 2}{2}(thb - tha)$.

Mihály Bencze

PP. 17192. If $x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\pi}{2} + arctg\left(\frac{\sin x + \cos x}{1 - \sin x \cos x}\right) \geq \frac{3\ln 2}{2\left(2+\sin^2 x \cos^2 x\right)}$.

Mihály Bencze

PP. 17193. If $x_k > 1$ $(k = 1, 2, ..., n)$, then $\prod_{k=1}^{n} \log_{x_k} \frac{S-x_k}{n-1} \geq 1$, where $S = \sum_{i=1}^{n} x_i$.

Mihály Bencze

PP. 17194. If $A_k \in P(M)$ $(k = 1, 2, ..., n)$, and $x_{ij} > 0$ such that $x_{i1} + x_{i2} + ... + x_{ii} = 1$ $(i, j = 1, 2, ..., n)$, then
$$\sum_{k=1}^{n} card(A_1 \cap A_2 \cap ... \cap A_k) \leq \sum_{i=1}^{n} \left(\sum_{j=i}^{n} x_{ji} card A_i \right).$$

Mihály Bencze

PP. 17195. Solve in N the equation
$$x_1 + \sqrt{x_2} + \sqrt[3]{x_3} + ... + \sqrt[n]{x_n} = \sqrt[n!]{x_1 + x_2 + ... + x_n}.$$

Mihály Bencze

PP. 17196. Determine all $n \in N^*$ such that $\frac{1}{\sin\left(x+\frac{\pi}{n}\right)} - \frac{1}{\sin\left(x-\frac{\pi}{n}\right)} \geq \frac{(n+1)\sqrt{n}}{n}$ for all $x \in \left[0, \frac{\pi}{2}\right)$.

Mihály Bencze

PP. 17197. Determine all $x, y, z \in R$ such that $\left|\frac{x+y+z}{1+xy+yz+zx}\right| \leq 1$, $\left|\frac{x-y+z}{1-xy+yz+zx}\right| \leq 1$, $\left|\frac{x+y-z}{1+xy-yz+zx}\right| \leq 1$, $\left|\frac{-x+y+z}{1+xy+yz-zx}\right| \leq 1$.

Mihály Bencze

PP. 17198. Compute $S_k = \left[k^{\sqrt{1}}\right] + \left[k^{\sqrt{2}}\right] + ... + \left[k^{\sqrt{n}}\right]$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 17199. Determine all $\lambda > 0$ for which the function $f : N^* \to N$, where $f(n) = [\lambda n(n+1)...(n+k)]$ is injective, where $k \in N^*$ is given and $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 17200. Determine all $a, b > 0$ such that
$arctg\frac{ax-by}{y\sqrt{a+b}} + arctg\frac{ay-bx}{x\sqrt{a+b}} = \frac{\pi}{a+b}$ for all $x, y > 0$.

Mihály Bencze

PP. 17201. If $a_k, b_k > 0$ $(k = 1, 2, ..., n)$ and $\{a_1, a_2, ..., a_n\} = \{b_1, b_2, ..., b_n\}$, then determine all $x, y > 0$ such that $\prod_{k=1}^{n} \left(a_k^x + a_k^y\right) \geq \prod_{k=1}^{n} \left(a_k^x + b_k^y\right)$.

Mihály Bencze

PP. 17202. Solve the equation $\sum_{k=1}^{n} \left[\frac{kx+k+1}{2k+1}\right] = n\left[\frac{(n+1)x+n+3}{2(n+2)}\right]$, where $[\cdot]$ denote the integer part.

Mihály Bencze

PP. 17203. Prove that
$2\left(arctg\frac{1}{2} + arctg\frac{1}{3}\right) = \arcsin\frac{4}{5} + \arcsin\frac{5}{13} + \arcsin\frac{16}{65}$.

Mihály Bencze