# Open questions

**OQ. 3544.** Let be $S(\alpha, p, n) = \frac{1}{n} \sum_{k=1}^{n} (k(k+1)...(k+p))^{\alpha}$. Compute the first decimal number after the decimal point of the numbers $S\left(\frac{1}{2}, p, n\right)$, $S\left(\frac{1}{3}, p, n\right)$, and in general case of $S(\alpha, p, n)$ if $\alpha \in Q$.

Mihály Bencze

**OQ. 3545.** If $a_0 = 3, a_1 = 2, a_2 = 1$, and $a_{n+2} = a_{n+1} + a_n + a_{n-1}$ for all $n \geq 1$, then determine all $k, p \in N$ such that $(a_k)^p - 1$ is divisible by $a_p$.

Mihály Bencze

**OQ. 3546.** Let $F(n)$ be the number of permutations $\{a_1, a_2, ..., a_n\}$ of the set $\{1, 2, ..., n\}$ such that $a_1 = 1$ and $|a_i - a_{i-1}| \leq k$ for $1 \leq i \leq n-1$, where $k \in N$, $k \geq 2$.
Determine all $p \in N$, such that $F(p)$ is divisible by $k+1$.

Mihály Bencze

**OQ. 3547.** Let be $k \in N$, $2 \leq k \leq n-1$

1). For every integer $n \geq k+1$, there exist $n$ distinct positive integers such that the product of any $k$ of them is divisible by the sum of the remaining $n-k$ numbers.

2). For some integers $n \geq k+1$, there exist $n$ distinct positive integers such that the sum of any $n-k$ of them is divisible by the product of the remaining $k$ numbers.

Mihály Bencze

**OQ. 3548.** Let $M = \left\{x^k + x^{k-1} + ... + x^2 + x \mid \text{is a positive integer}\right\}$.

Determine all $k \in N$ such that for each integer $p \geq k$ there exists $a_1, a_2, ..., a_p, b_p$ in $M$ such that $a_1 + a_2 + ... + a_p = b_p$.

Mihály Bencze

**OQ. 3549.** Let $n \geq 2$ be an integer and let $M$ be a set with $n+1$ elements. The sequences $\{a_1, a_2, ..., a_n\}$ ; $\{b_1, b_2, ..., b_n\}$ , $\{c_1, c_2, ..., c_n\}$ of distinct elements from the set $M$ are called $\lambda$- separated if $a_i = \lambda b_j + (1-\lambda) c_k$ for some $i \neq j \neq k$, where $\lambda \in [0,1]$ . Determine the maximum number of ordered sequences of $n$ elements from $M$ such that any three of them are $\lambda$-separated.

Mihály Bencze

**OQ. 3550.** If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\sum\limits_{k=1}^{n} a_k = 1$, then

1). $\sum\limits_{k=1}^{n} (1 + a_k) \sqrt{\frac{1}{a_k} - 1} \geq \frac{\left(\sqrt{n-1}\right)^{n+1} n^{\frac{n+2}{2}}}{(n+1)^{n-1}} \prod\limits_{k=1}^{n} \frac{1+a_k}{\sqrt{1-a_k}}$

2). $\sum\limits_{k=1}^{n} (1 + a_k) \left(\frac{1}{a_k} - 1\right)^{\alpha} \geq \frac{(n-1)^{(n+1)\alpha}}{n^{(n-1)\alpha}(n+1)^{n-1}} \prod\limits_{k=1}^{n} \frac{1+a_k}{(1-a_k)^{\alpha}}$,
where $\alpha \in [0,1]$ .

Mihály Bencze

**OQ. 3551.** If $x_k > 0$ $(k = 1, 2, ..., n)$ , then

1). $\left(\frac{\sum\limits_{k=1}^{n} x_k^n}{\prod\limits_{k=1}^{n} x_k}\right)^{\frac{1}{n}} + \left(\frac{\sum\limits_{1 \leq i < j \leq n} x_i x_j}{\sum\limits_{k=1}^{n} x_k^2}\right)^{\frac{1}{2}} \geq \sqrt[n]{n} + \sqrt{\frac{n-1}{2}}$

2). $\left(\frac{\sum\limits_{i=1}^{n} x_i^n}{\prod\limits_{i=1}^{n} x_i}\right)^{\frac{1}{n}} + \left(\frac{\sum\limits_{1 \leq i_1 < ... < i_k \leq n} x_{i_1} x_{i_2} ... x_{i_k}}{\sum\limits_{i=1}^{n} x_i^k}\right)^{\frac{1}{k}} \geq \sqrt[n]{n} + \left(\frac{1}{n}\binom{n}{k}\right)^{\frac{1}{k}}$
for all $k \in \{1, 2, ..., n\}$

Mihály Bencze

**OQ. 3552.** Compute $S(\alpha, n, k) = \sum\limits_{1 \leq i_1 < ... < i_k \leq n} \frac{1}{i_1 i_2 ... i_k \left(i_1^{\alpha} + i_2^{\alpha} + ... + i_k^{\alpha}\right)}$ for $\alpha = 1, 2, ...$ and $k \in N^*$.

Mihály Bencze

**OQ. 3553.** If $x_k > 0$ $(k = 1, 2, ..., n)$ and $\prod\limits_{k=1}^{n} x_k = 1$, then

$$(n-1)\sqrt{n\sum_{cyclic} x_1^{x_2}} + \sum_{k=1}^{n} x_k \geq n^2.$$

Mihály Bencze

**OQ. 3554** If $a_k > 0$ $(k = 1, 2, ..., n)$, then determine all $x, y \in R$ for which

$$\min\left\{a_1^x - a_2^y; a_2^x - a_3^y; ...; a_n^x - a_1^y\right\} \leq \frac{1}{n}.$$

Mihály Bencze

**OQ. 3555.** Let $A_1A_2...A_n$ be a simplex which contains the centre $O$ of its circumsphere as an interior point. Let $d_i$ be the distance from $O$ to the face opposite vertex $A_i$ $(i = 1, 2, ..., n)$. If $R$ denote the radius of the circumsphere, prove that $\sum\limits_{i=1}^{n} d_i \leq \sqrt[n]{n}R$.

Mihály Bencze

**OQ. 3556.** If $0 < x_1 \leq x_2 \leq ... \leq x_n$, then determine all $\alpha > 0$ and all $n \in N$ for which $\sum\limits_{cyclic} \ln x_1 \ln\left(\frac{\alpha + x_2}{\alpha + x_3}\right) \geq 0$.

Mihály Bencze

**OQ. 3557.** Determine all $x > 0$ such that $x(ay+1)^{-\alpha} + y(a + x^{\alpha})^{-\alpha} \leq (x+1)(a+1)^{-\alpha}$ for all $y > 0$ where $\alpha \in [0, 1]$ and $a > 0$ are given.

Mihály Bencze

**OQ. 3558.** If $x_i > 0$ $(i = 1, 2, ..., n)$, then

$$\prod_{1 \leq i_1 < ... < i_k \leq n} (x_{i_1} + ... + x_{i_k}) \geq n\left(\prod_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} x_i\right)^{\binom{n}{k} - n}.$$

Mihály Bencze

**OQ. 3559.** Let $f: R \to R$ be a function such that $f(x) = \sum\limits_{i=0}^{\infty} a_i x^i$. Find the sums:

1). $S_\alpha = \sum\limits_{n=1}^{\infty} (-1)^n \left(f^\alpha(x) - a_0^\alpha - (a_1 x)^\alpha - \dots - \left(a_{n+k} x^{n+k}\right)^\alpha\right)$, where $\alpha \in R$

2). $P_\alpha = \sum\limits_{n=1}^{\infty} (-1)^n \left(f(x) - a_0 - a_1 x - \dots - a_{n+k} x^{n+k}\right)^\alpha$

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**OQ. 3560.** If $x_k > 0$ $(k = 1, 2, ..., n)$ such that $\sum\limits_{k=1}^{n} x_k = 1$, then

$\sum\limits_{k=1}^{n} \frac{x_k + a}{x_k^\alpha + b} \le \frac{(na+1)n^\alpha}{1+bn^\alpha}$, where $a, b > 0$ and $\alpha \ge 1$.

Mihály Bencze

**OQ. 3561.** If $a, b, x_i > 0$ $(i = 1, 2, ..., n)$, then

$a \sum\limits_{i=1}^{n} x_i + b \sqrt[k]{\sum\limits_{i=1}^{n} x_i^k} \le \frac{na + b\sqrt[k]{n}}{n^k} \left(\sum\limits_{i=1}^{n} \frac{1}{x_i}\right)^{k-1} \left(\sum\limits_{i=1}^{n} x_i^k\right)$, when $k \in N^*$.

Mihály Bencze

**OQ. 3562.** If $n, p \in N$ $(p \ge 2)$, then $\sum\limits_{k=0}^{(p-1)n-1} n^{\frac{1}{n+k}} \ge n\sqrt[n]{p}$.

Mihály Bencze

**OQ. 3563.** Determine all $a_k > 0$ $(k = 1, 2, ..., n)$ such that
$\sum\limits_{k=1}^{n} a_k^2 \ge \frac{n^2 \prod\limits_{cyclic} \left(a_1^2 + a_1 a_2 + a_2^2\right)}{3^n \sum\limits_{cyclic} a_1^{n-1} a_2^{n-1}} \ge \sum\limits_{cyclic} a_1 a_2$.

Mihály Bencze

**OQ. 3564.** If $a_{ij} > 0$ $(i = 1, 2, ..., n; j = 1, 2, ..., m)$, then
$n\left(\sum\limits_{i=1}^{n} \prod\limits_{j=1}^{m} a_{ij} + \sqrt[m]{\prod\limits_{j=1}^{m} \left(\sum\limits_{i=1}^{n} a_{ij}^m\right)}\right) \ge 2 \prod\limits_{j=1}^{m} \left(\sum\limits_{i=1}^{n} a_{ij}\right)$.

Mihály Bencze

**OQ. 3565.** Denote $S_k(n)$, the number of all $n$ - digit numbers in base 10 each of which contains the block $\overline{a_1a_2...a_k}$ $(a_1, a_2, ..., a_k \in \{0, 1, ..., 9\})$ and is divisible by $\overline{a_1a_2...a_k}$

1). Determine $S_k(n)$

2). Compute $\sum\limits_{n=1}^{\infty} \frac{1}{S_k^{\alpha}((n))}$, where $\alpha \in R$

Mihály Bencze

**OQ. 3566.** Let be $M(x) = \{[kx] \mid k = 1, 2, ...\}$, and $x_i > 1$ $(i = 1, 2, ..., n)$ such that $\sum\limits_{i=1}^{n} \frac{1}{x_i} > 1$. Prove that there exist $1 \le i_1 < i_2 \le ... \le i_{n-1} \le n$, such that $S(x_{i_1}) \cap S(x_{i_2}) \cap ... \cap S(x_{i_{n-1}})$ has infinitely many elements, where $[\cdot]$ denote the integer part.

Mihály Bencze

**OQ. 3567.** Determine all $n, k \in N$ $(n < k)$ for which

$a^n + b^n + (a-b)^n = c^n + d^n + (c-d)^n$ implies
$a^k + b^k + (a-b)^k = c^k + d^k + (c-d)^k$, for all $a, b, c, d \ge 0$.

Mihály Bencze

**OQ. 3568.** If $a^n + b^n = (a+b)^n + c_1\ ab(a+b)^{n-2} + c_2a^2b^2(a+b)^{n-4} + ...$
Then compute

1). $c_1^k + c_2^k + ...$

2). $\frac{1}{c_1^k} + \frac{1}{c_2^k} + ...$ where $k \in N$.

From Waring formula we have $c_1 = -n, c_2 = \frac{n(n-3)}{2} etc.$

Mihály Bencze

**OQ. 3569.** If $x(x+1)...(x+m) = a_{m+1}x^{m+1} + a_mx^m + ... + a_1x$, then $a_1 = m!$, $a_m = \frac{m(m+1)}{2}$, $a_{m+1} = 1$. Compute:

1). $a_1^k + a_2^k + ... + a_{m+1}^k$

2). $\sum\limits_{m=1}^{\infty} \frac{1}{a_m^k}$, where $k \in N$.

Mihály Bencze

**OQ. 3570.** Compute:

1). $S(k) = \sum\limits_{n=1}^{\infty} \frac{1}{(k+n)^n}$, where $k \in N$.

We have $S(0) = \sum\limits_{n=1}^{\infty} \frac{1}{n^n} = \int\limits_0^1 x^{-x} dx$.

2). $P(k) = \sum\limits_{n=1}^{\infty} \frac{1}{k+n^n}$

Mihály Bencze

**OQ. 3571.** Determine all $a_k, b_k, c_k, d_k \in R$ $(k = 1, 2, ..., n)$ such that the functions $f(x) = \left\{ x^n + \sum\limits_{k=1}^{n} a_k x^{n-k} + \sin x \right\}$,

$g(x) = \left\{ x^n + \sum\limits_{k=1}^{n} b_k x^{n-k} + \cos x \right\}$, $h(x) = \left\{ x^n + \sum\limits_{k=1}^{n} c_k x^{n-k} + tgx \right\}$,

$r(x) = \left\{ x^n + \sum\limits_{k=1}^{n} d_k x^{n-k} + ctgx \right\}$ are periodic, when $\{\cdot\}$ denote the fractional part.

Mihály Bencze

**OQ. 3572.** Do there exist two sequences of real numbers $(a_n)_{n\geq 1}$, $(b_n)_{n\geq 1}$ such that $0 \leq a_n \leq b_n \leq \frac{\pi}{2}$ and $\sin(a_n x) - \cos(b_n x) \leq \frac{1}{n}$ for all $n \in N^*$ and $0 < x < 1$?

Mihály Bencze

**OQ. 3573.** If $P$ is a polynomial with real coefficients such that

$(xP(y) + yP(x))^2 = x^2 y^2 P(xy)$ for all $x, y \in R$ there exist $x_0 \in R$ such that $P(x_0) = 0$.

Mihály Bencze

**OQ. 3574.** If $T_k(n) = \frac{n(n+1)...(n+k-1)}{k}$, then compute $\sum\limits_{n=1}^{\infty} \frac{1}{p+(T_k(n))^r}$, where $p \in Z^*$ and $r \in R$ are given.

Mihály Bencze

**OQ. 3575.** Find all function $f : R \to R$ such that

$$\prod_{k=1}^{n} f(x_k) = f\left(\sum_{k=1}^{n} x_k\right) + \prod_{k=1}^{n} x_k \text{ for all } x_k \in R \ (k = 1, 2, ..., n).$$

Mihály Bencze

**OQ. 3576.** If $a_k \in R$ $(k = 1, 2, ..., n)$, then find all polynomials $P(x) = x^n + \sum_{k=1}^{n} a_k x^{n-k}$ for which $-\varepsilon \le P(x) \le \varepsilon$ for all $x \in R$, where $\varepsilon > 0$ is given.

Mihály Bencze

**OQ. 3577.** Find all functions $f : [0, +\infty) \to [0, +\infty)$ such that

$$f\left(\prod_{i=1}^{n} x_i\right) + \sum_{i=1}^{n} f(x_i) = \prod_{1 \le i_1 < ... < i_k \le n} f\left(\sqrt[k]{x_{i_1} x_{i_2} ... x_{i_k}}\right) \text{ for all } x_i \ge 0$$
$(i = 1, 2, ..., n)$.

Mihály Bencze

**OQ. 3578.** If $a_k \in N$ $(k = 1, 2, ..., n)$, then $\sum_{k=1}^{n} \left(a_k + \frac{1}{n}\right)^p$ is an integer for only finitely many positive integers $p$.

Mihály Bencze

**OQ. 3579.** The function $a : N \to N$ is defined for non-negative integers $n$, $a(n)$ is equal to the sum of the perfect $k$ power of the digits of the decimal representation of $n$. Thus for the example $a(12) = 1^k + 2^k$.
1). Find all $p, n$ such that $a(n) = n^p$
2). Find all $p, n$ such that $a(pn) = n^{p+1}$

Mihály Bencze

**OQ. 3580.** Let $p \ge 3$ be a prime number, and $k, n \in N^*$. How many $p$-element subsets $M$ of $\{k, k+1, ..., pn\}$ are there, the sum of whose elements is divisible by p?

Mihály Bencze

**OQ. 3581.** Let be $a_1 > 1$. Determine all $k, p \in N$ for which exist an increasing sequence of positive integers $a_1, a_2 ...$ such that $\sum\limits_{i=1}^{n} a_i^k$ dividet $\sum\limits_{i=1}^{p} a_i^p$ for all $n \in N$.

Mihály Bencze

**OQ. 3582.** Let $A_k$ $(k = 1, 2, ..., n)$ be sets of $N$ residues (mod $N^n$). Determine all $n \in N$ such there exists a set $A_n$ of $N$ residues (mod$N^n$) such $\sum\limits_{k=1}^{n} A_k = \left\{ \sum\limits_{k=1}^{n} x_k | x_1 \in A_1, x_2 \in A_2, ..., x_n \in A_n \right\}$ contains at least $\frac{1}{n}$ of all the residues (mod $N^n$).

Mihály Bencze

**OQ. 3583.** Determine all $k \in N$ for which $(\sigma(n))^k < n^{k+1}$ for all $n$ odd positive integers.

Mihály Bencze

**OQ. 3584.** 1). Prove that are infinitely many $n$ such that $1^{1!} + 2^{2!} + ... + n^{n!}$ is composite.
2). Determine all $n \in N$ for which $1^{1!} + 2^{2!} + ... + n^{n!}$ is prime.

Mihály Bencze

**OQ. 3585.** Let $p$ be a prime. Find all positive integers $k$ and $n$ such that the set $\{1^k, 2^k, ..., n^k\}$ can be partitioned into $p$ subsets with equal sum of elements.

Mihály Bencze

**OQ. 3586.** Find the maximum number of pairwise disjoint sets of the form $\{n^k + a_1 n^{k-1} + ... + a_{k-1} n + a_k | n \in Z\}$ with $a_i \in Z$ $(i = 1, 2, ..., k)$.

Mihály Bencze

**OQ. 3587.** Prove that if $\alpha_k$ $(k = 1, 2, ..., n)$ are positive irrational numbers such that $\sum\limits_{k=1}^{n} \frac{1}{\alpha_k} = 1$, then the sequences $[\alpha_k], [2\alpha_k], [3\alpha_k], ...$ $(k = 1, 2, ..., n)$ together include every positive integer exactly once, where $[\cdot]$ denote the integer part.

Mihály Bencze

**OQ. 3588.** Determine all $\alpha, \beta \in R$ such that $\Phi(n) \leq n^{\alpha} - n^{\beta}$ for all $n$ composite positive integers.

Mihály Bencze

**OQ. 3589.** Determine all $n > k > 1$ $(n, k \in N)$ for which there are $a_i > 1$ $(i = 1, 2, ..., n)$, such that $\sum_{i=1}^{n} a_i = n$, and $\sum_{i=1}^{n} \frac{\Phi(a_i)}{a_i} > 1$.

Mihály Bencze

**OQ. 3590.** Determine all $n, k$ for which $\sigma(n) - d(k)$ is even.

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**OQ. 3591.** Find all $x, y, n \in N$ such that $\frac{x^n+y}{y^n-x}$ and $\frac{y^n+x}{x^n-y}$ are both integers.

Mihály Bencze

**OQ. 3592.** Find all $x_k \in N$ $(k = 1, 2, ..., n)$ for which
$(\gcd(x_1, x_2, ..., x_k))^k = x_1 + x_2 + ... + x_k, (\gcd(x_2, x_3, ..., x_{k+1}))^k =$
$x_2 + x_3 + ... + x_{k+1}, ..., (\gcd(x_n, x_1, ..., x_{k-1}))^k = x_n + x_1 + ... + x_{k-1}$.

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**OQ. 3593.** Determine all $r \in N$ for which $\sum_{k=0}^{p} \binom{p}{k}\binom{p+k}{k} \equiv (r^p + 1) \pmod{p^r}$.

Mihály Bencze

**OQ. 3594.** Determine all $k, p \in N$ for which $n^{\pi(kn)-\pi(pn)} < \left(\frac{k}{p}\right)^{\frac{kn}{p}}$ for all positive integer $n$.

Mihály Bencze

**OQ. 3595.** For any positive integer $n > 1$, let $\alpha(n)$ be the greatest prime divisor of $n$. Determine all $x_k, y_k, z_k \in N$ $(k = 1, 2, )$, $x_k < y_k < z_k$ for which are infinitely many $n, m \in N$ such that $\alpha(n + x_1) < \alpha(n + y_1) < \alpha(n + z_1)$ and $\alpha(m + x_2) > \alpha(m + y_2) > \alpha(m + z_2)$.

Mihály Bencze

**OQ. 3596.** Find all positive integers, representable uniquely as $\frac{x^n+y}{xy+n-1}$, where $x, y, n \in N$.

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**OQ. 3597.** Determine all $x, y, n, m \in N$ such that $x^m + y^m$ divides $(x+y)^n$.

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**OQ. 3598.** Determine all $k, p, r \in N$ for which $(n+r)(n+r+1)...(n+k)$ is not a perfect $p$ power, for all $n \in N$.

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**OQ. 3599.** Determine all $k \in N$ for which exist infinitely many composite numbers $n$ such that $(k+1)^{n-1} - k^{n-1}$ is divisible by $n$.

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**OQ. 3600.** Determine all $r, a_k \in N$ $(k = 1, 2, ..., n)$ such that $\sum\limits_{k=1}^{n} a_k$ divides $\sum\limits_{k=1}^{n} a_k^r$ and there are infinitely many positive integers $m$ such that $\sum\limits_{k=1}^{n} a_k$ divides $\sum\limits_{k=1}^{n} a_k^m$.

Mihály Bencze

**OQ. 3601.** Let $p > 3$ be a prime number. Determine all $r \in N$ for which $\binom{p}{1} + \binom{p}{2} + ... + \binom{p}{k}$ is divisible by $p^r$ where $k = \left[\frac{pr}{r+1}\right]$, and $[\cdot]$ denote the integer part.

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**OQ. 3602.** Determine all $p, k \in N$ for which $lcm(p, p+1, ..., kn)$ is divisible by $\binom{kn}{pn}$ for all $n \in N^*$.

Mihály Bencze

**OQ. 3603.** Determine all $\alpha \in R$ for which exist infinitely many positive integers $n$ such that $[n^\alpha] + 1$ divides $n!$, where $[\cdot]$ denote the integer part.

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**OQ. 3604.** Let $p_k$ $(k = 1, 2, ..., n)$ distinct primes greater than 3. Determine all $r \in N$ $(r \geq 2)$ for which $r^{\prod\limits_{k=1}^{n} p_k} + 1$ has at least $r^{2n}$ divisors.

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**OQ. 3605.** Find all $p, x_k \in N$ $(k = 1, 2, ..., n)$ for which $\prod\limits_{cyclic} (x_1 x_2 ... x_{n-1} + p)$ is a perfect $n - 1$ power if and only if

$x_1 x_2 ... x_{n-1} + p, x_2 x_3 ... x_n + p, ..., x_n x_1 ... x_{n-2} + p$ are perfect $n - 1$ powers.

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**OQ. 3606.** Determine all $p, x_k \in N$ $(k = 1, 2, ..., n)$ such that $\prod\limits_{k=1}^{n} x_k + p^n$ divides $\sum\limits_{k=1}^{n} x_k^n$, then $\frac{\sum\limits_{k=1}^{n} x_k^n}{\prod\limits_{k=1}^{n} x_k + p^n}$ is a perfect $n$ power.

Mihály Bencze

**OQ. 3607.** Determine all $a_k \in N$ $(k = 1, 2, ..., n)$, for which $\prod\limits_{1 \leq i < j \leq n} (a_j - a_i)$ is divisible by $n(n-1)$.

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**OQ. 3608.** Determine all $r, n_k, p \in N$ $(k = 1, 2, ..., n)$, for which $p^n + \prod\limits_{k=1}^{n} n_k$ is divisible by $r!$ then $\sum\limits_{k=1}^{n} n_k$ is divisible by $r!$.

Mihály Bencze

**OQ. 3609.** 1). Let $\alpha(n)$ be the greatest odd divisors of n. Prove that $\frac{1}{p^n} \sum\limits_{k=1}^{p^n} \frac{\alpha(k)}{k} > \frac{p}{p+1}$, where $p \geq 2$, $p \in N$.

2). Determine the minimum of the expression $\frac{1}{p^n} \sum\limits_{k=1}^{p^n} \frac{(\alpha(k))^x}{k^y}$, where $x, y \in R$

Mihály Bencze

**OQ. 3610.** Find all $a_k \in N$ $(k = 1, 2, ..., n+1)$ such that $p^{a_{n+1}} - r$ divides $r + \sum\limits_{k=1}^{n} p^{a_k}$, where $p, r \in N$ are giving.

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**OQ. 3611.** Find all integers $k, a_i \in N$ $(i = 1, 2, ..., n)$ with $1 < k < a_1 < a_2 < ... < a_n$ such that $\prod\limits_{i=1}^{n} (a_i - k)$ is a divisor of $\prod\limits_{i=1}^{n} a_i - k^n$.

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**OQ. 3612.** 1). Prove that $\int\limits_0^1 \frac{\ln x\left(1+\ln^2 x\right)}{1-x} dx = -\frac{\pi^2}{6} - \frac{\pi^4}{15}$

2). Compute $\int\limits_0^1 \frac{\ln x(1+\ln x)\left(1+\ln^2 x\right)...\left(1+\ln^k x\right)}{1-x} dx$

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**OQ. 3613.** Let $p \geq 3$ be a prime number. Determine the greatest common divisor of the elements of the set $\{n^p - n | n \in Z\}$.

Mihály Bencze

**OQ. 3614.** 1). Prove that $\int\limits_0^1 \ln\left((1+x)\left(1-x^2\right)\right) \ln x dx = 6 - 4\ln 2 - \frac{\pi^2}{3}$

2). Compute $\int\limits_0^1 \ln\left((1+x)\left(1-x^k\right)\right) \ln x dx$, where $k \in N^*$.

Mihály Bencze

**OQ. 3615.** 1). Prove that $\int\limits_0^1 \frac{\ln x dx}{1-x^2} = -\frac{\pi^2}{8}$

2). Compute $\int\limits_0^1 \frac{\ln x dx}{1-x^k}$, where $k \in N^*$

2). Compute $\int\limits_0^1 \frac{\ln x dx}{\prod\limits_{k=1}^{n}\left(1-x^k\right)}$

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**OQ. 3616.** 1). If $a,b,c\in Z$ such that $a^2+b^2=c^2$, then $a^3+b^3+c^3$ is composite

2). If $a_k,b\in Z$ $(k=1,2,...,n)$ such that $\sum\limits_{k=1}^{n}a_k^2=b^2$, then determine all $n\in N$ for which $b^3+\sum\limits_{k=1}^{n}a_k^3$ is composite

3). If $a_k,b\in Z$ $(k=1,2,...,n)$ such that $\sum\limits_{k=1}^{n}a_k^x=b^x$, then determine all $n,x,y\in N$ for which $b^{x+y}+\sum\limits_{k=1}^{n}a_k^{x+y}$ is composite

4). If $a,b,c\in N$ such that $a^2+b^2=c^2$, then determine all $n\in N$ for which $a^n+b^n+c^n$ is prime

5). If $a_k,b\in N$ $(k=1,2,...,n)$ such that $\sum\limits_{k=1}^{n}a_k^2=b^2$, then determine all $m\in N$ for which $b^m+\sum\limits_{k=1}^{n}a_k^m$ is prime.

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**OQ. 3617.** Let $(x,<>)$ be a real inner product space, and let $B=\{x\in X|\,\|x\|\leq a\}$, when $\|x\|=\sqrt{\langle x,x\rangle}$. Let $f:B\to B$ be a function satisfying $\|f(x)-f(y)\|\leq b\|x-y\|$ for all $x,y\in B$. Determine all $a,b>0$ for which the set of fixed points of $f$ is convex.

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**OQ. 3618.** Determine the best constants $a,b>0$ such $\sum\limits_{k=1}^{n}\frac{1}{k}=a\ln(n+b)+\gamma+\sum\limits_{k=1}^{\infty}\frac{\left(1-\frac{1}{2^{2k-1}}\right)B_{2k}}{2k(n+b)^{2k}}$, where $B_{2k}$ are the Bernoulli numbers.

Mihály Bencze

**OQ. 3619.** Compute

1). $\sum\limits_{k=1}^{\infty}\frac{(-1)^{F_k}}{k}$

2). $\sum\limits_{k=1}^{\infty}\frac{(-1)^{L_k}}{k}$

where $F_k$ and $L_k$ are the Fibonacci‘s and Luca‘s numbers.

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**OQ. 3620.** Determine all $\alpha, \beta$ such that
$$\sum_{cyclic}\left(\frac{a_1}{a_2}\right)^{\alpha}+\left(1+\frac{n\prod\limits_{k=1}^{n}a_k}{\sum\limits_{k=1}^{n}a_k^n}\right)^{\beta}\geq n^{\alpha}+2^{\beta} \text{ for all } a_k>0\ (k=1,2,...,n).$$

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**OQ. 3621.** Compute $S_\alpha=\lim\limits_{n\to\alpha}\frac{1}{n^\alpha}\sum\limits_{k=1}^{n-1}k\left(\left[x+\frac{n-k-1}{n}\right]\right)^{\alpha-1}$ where $[\cdot]$ denote the integer part. We have $S_2=\frac{1}{2}\left([x]+(\{x\})^2\right)$.

Mihály Bencze

**OQ. 3622.** Let $M$ be a largest subset of $\{1,2,...,n\}$ such that each elements of $M$ divides at most one other element of $M$. Determine cardinal $M$. Determine all $a,b\in Q$ such that $cardM=an+b$.

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**OQ. 3623.** If $a,b,c>0$, then $\sum\frac{a}{b+c}+\sum\sqrt{\frac{bc}{(a+b)(a+c)}}\geq 3$.

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**OQ. 3624.** Determine all $a_k\in Z$ $(k=1,2,...,n)$ such that $(n+1)!=\sum\limits_{k=1}^{n}a_k k!$.

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**OQ. 3625.** Solve in $Z$ the equation $\sum\limits_{k=1}^{n}\frac{k!}{x_k}=\frac{1}{(n+1)!}$.

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**OQ. 3626.** 1). Prove that exist infinitely many $a_k\in C$ $(k=0,2,...,n)$ such that $\sum\limits_{k=0}^{n}a_k\binom{n}{k}^p=0$, where $p\in N$

2). Determine all $b_k\in C$ $(k=0,1,...,n)$ such that $\sum\limits_{k=0}^{n}b_k=\binom{2n}{n}$ and $\sum\limits_{k=0}^{n}b_k\binom{n}{k}^2=0$.

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**OQ. 3627.** If $A \in M_n(C)$ then determine all $a_k \in C$ $(k = 1, 2, ..., n)$ such that $\det\left(\sum_{k=1}^{n} a_k A^{k-1}\right) = \sum_{k=1}^{n} a_k \det\left(A^{k-1}\right).$

Mihály Bencze

**OQ. 3628.** Determine all $a_k, b_k > 0$ $(k = 1, 2, ..., n)$ such that

$\left(\frac{1}{n}\sum_{k=1}^{n} \frac{a_k^x}{b_k^y}\right)\left(\frac{1}{n}\sum_{k=1}^{n} b_k\right)^y \geq \left(\frac{1}{n}\sum_{k=1}^{n} a_k\right)^x$ for all $x, y \geq 0$.

Mihály Bencze

**OQ. 3629.** Solve in $N$ the equation $\sum_{k=1}^{n} x_k^3 = y^2$. (A solution is $x_k = kp^2$ $(k = 1, 2, ..., n)$, $y = \frac{n(n+1)}{2}p^3$, $p \in N$).

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**OQ. 3630.** Determine all $a_1, a_2, ..., a_k \in R$ for which the sequence $x_n = (-1)^{[na_1]} + (-1)^{[na_2]} + ... + (-1)^{[na_k]}$ is periodical, for all $n \geq 1$. ($[\cdot]$ denote the integer part).

Mihály Bencze

**OQ. 3631.** 1). Denote $S(n)$ the sum of digits of number $n$.
Determine all $k \in N^*$ such that for any $n_0 \in N^*$ there is $n \geq n_0$ $(n \in N)$ such that $S(k^n) \geq S\left(k^{n+1}\right)$

2). Determine all $a, b, c \in N^*$ such that $S^2(a) + S^2(b) = S^2(c)$

3). Compute $\sum_{n=1}^{\infty} \frac{S(n)}{n(n+1)...(n+k)}$.

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**OQ. 3632.** Denote $p_k$ the $k$-th prime $(p_1 = 2)$. Determine all $p_i$ $(i = 1, 2, ..., n)$ for which $\prod_{i=1}^{n} p_i = \overline{a_1a_2...a_raa_1a_2...a_r}$, where $a, a_j \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $(j = 1, 2, ..., r)$.
A solution is $7 \cdot 11 \cdot 13 \cdot 17 = 17017$.

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**OQ. 3633.** Denote $p_k$ the $k$-th prime $(p_1 = 2)$, and

$S(n,k) = \frac{1}{p_1 p_2 \cdots p_k} + \frac{1}{p_2 p_3 \cdots p_{k+1}} + ... + \frac{1}{p_n p_{n+1} \cdots p_{n+k-1}}.$

Determine the integer part of $2^k S(n,k)$ for all $n, k \in N$ $(k \geq 2)$.

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**OQ. 3634.** Determine all $k, p \in N$ for which $\prod\limits_{1 \leq i < j \leq n} \frac{a_i^k - a_j^k}{i^p - j^p}$ is an integer for all integers $a_1, a_2, ..., a_n$.

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**OQ. 3635.** Determine all $k \in N$ for which $\prod\limits_{i=1}^{k} (n+i-1)!$ dividet $(kn)!$.

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**OQ. 3636.** Determine all $k \in N^*$ for which $\prod\limits_{i=0}^{n-1} \left(k^n - k^i\right)$ is divisible by $(n!)^{k-1}$.

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**OQ. 3637.** Solve in natural numbers the equation

$\frac{1}{k^m} = \pm \frac{1}{n_1!} \pm \frac{1}{n_2!} \pm ... \pm \frac{1}{n_r!}.$

Mihály Bencze

**OQ. 3638.** If $S_n^k = 1^k + 2^k + ... + n^k$, then determine the best constant $c_k > 0$ such that $1 + \sum\limits_{i=2}^{n} \sqrt[k]{\frac{S_n^k - S_{i-1}^k}{i}} \leq c_k S_n^{k-1}$ for all $n \geq 1$.
If $k = 2$ then $c_2 = \pi$.

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**OQ. 3639.** Prove that if $a_{ij} > 0$ $(i = 1, 2, ..., n; j = 1, 2, ..., k)$, then

$\prod\limits_{j=1}^{k} \left( \sum\limits_{1 \leq i_1 < ... < i_k \leq n} \min \{a_{i_1 j}; a_{i_2 j}; ...; a_{i_k j}\} \right) \geq$

$$\geq \left( \sum_{1 \leq i_1 < ... < i_k \leq n} \min \{a_{i_1 1}; a_{i_2 2}, ..., a_{i_k k}\} \right)^{\alpha} \text{ is true for } \alpha = 1.$$

What is the best possible value of $\alpha > 0$ for which holds the given inequality. What happens when min is replaced by max?

Mihály Bencze

**OQ. 3640.** Determine all $m, k, p \in N$ for which exist $n_1, n_2, ..., n_r \in N$ such that $(n_1! - k)(n_2! - k) ... (n_r! - k) - p^m$ is a perfect square.

Mihály Bencze

**OQ. 3641.** Determine the best constant $c_k > 0$ such that

$$\sum_{1 \leq i_1 < ... < i_k \leq n} \frac{a_{i_1} a_{i_2} ... a_{i_k}}{i_1 + i_2 + ... + i_k} \leq c_k \sum_{i=1}^{n} a_i^k \text{ for all } a_i > 0 \ (i = 1, 2, ..., n).$$

If $k = 2$ then $c_k = \pi$ and we re obtain the Hilbert‘s inequality.

Mihály Bencze

**OQ. 3642.** 1). Denote $p_k$ the $k$-th prime $(p_1 = 2)$, and $x_n$ denote the first digit of $p_1^n + p_2^n + ... + p_k^n$.
Determine all $k$ for which the sequence $(x_n)_{n \geq 1}$ is not periodical.
2). Denote $y_n$ the first two digits of $p_1^n + p_2^n + ... + p_k^n$.
Determine all $k$ for which the sequence $(x_n)_{n \geq 1}$ is periodical.

Mihály Bencze

**OQ. 3643.** For every $k$ positive integers determine all $A, B, C$ irrational numbers such that on can find distinct positive integers $n_1, n_2, ..., n_k$ for which $[n_1 A], [n_2 A], ..., [n_k A]$ and $[n_1 B], [n_2 B], ..., [n_k B]$ and $[n_1 C], [n_2 C], ..., [n_k C]$ are geometrical sequences, where $[\cdot]$ denote the integer part.

Mihály Bencze

**OQ. 3644.** If $r_k \in (0, 1)$ $(k = 1, 2, ...)$ are a given sequence, then determine all $x > 0$ and all $y \in (0, 1)$ such that $\sum_{k=1}^{\infty} r_k^x = y$.

Mihály Bencze

**OQ. 3645.** 1). If $a, b, c, d \geq 5$ are prime numbers then the Vandermonde determinant $V(a, b, c, d)$ is divisible by 576. Because $a, b, c, d \geq 5$ are prime, then all have the form $6k \pm 1$. Therefore exist at least two from $a, b, c, d$ which difference is $6M$ and the another difference is even, therefore
$V(a, b, c, d) = (d-a)(d-b)(d-c)(c-a)(c-b)(b-a)$ is divisible by
$36 \cdot 16 = 576$

2). If $a_k \geq 5$ $(k = 1, 2, ..., n)$ are prime, then the Vandermonde determinant $V(a_1, a_2, ..., a_n)$ is divisible by $2^{r(n)}3^{s(n)}$. We have $r(2) = 2,\ s(2) = 2$.

In general we get $r(2) = 2,\ s(n) = \frac{n(n-1)}{2}$. What is the general form of $r(n)$ and $s(n)$?

3). What happens if $a_k \geq 3$ $(k = 1, 2, ..., n)$?

Mihály Bencze

**OQ. 3646.** If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\prod_{k=1}^{n} a_k = 1$, then determine all $\alpha, \beta > 0$ such that $\frac{1}{n}\sum_{k=1}^{n} a_k \geq \frac{1}{n}\left(\sum_{k=1}^{n} a_k^{\alpha}\right)^{\beta}$

If $n = 3$ then $\alpha = 2$ and $\beta = \frac{1}{5}$ is a solution.

Mihály Bencze

**OQ. 3647.** If $\alpha_k \in C \backslash R$ $(k = 1, 2, ..., p)$, and $|\alpha_k| \leq 1$ $(k = 1, 2, ..., p)$, then the equation $z^n + \alpha_1 z^{n-1} + ... + \alpha_p z^{n-p} + ... + \alpha_p z^p + ... + \overline{\alpha_2} z^2 + \overline{\alpha_1} z + 1 = 0$ have all roots of module 1.
If $k = 1$, then $z^n + \alpha_1 z^{n-1} + \overline{\alpha_1} z + 1 = 0$ is equivalent with $\left(|z|^{2(n-1)} - 1\right)|z + \alpha|^2 + \left(|z|^2 - 1\right)\left(1 - |\alpha|^2\right) = 0$. Because $|\alpha| < 1$, therefore $|z| = 1$. What happens if $|\alpha_k| > 1$ $(k = 1, 2, ..., n)$?

Mihály Bencze

**OQ. 3648.** If $a_k > 0$ $(k = 1, 2, ..., n)$ and $\prod_{k=1}^{n} a_k = 1$, then determine all $\alpha > 0$ such that $\sum_{cyclic} \frac{a_1 - \alpha}{a_2} \geq 0$ and $\sum_{cyclic} \frac{a_1 - \alpha}{a_2 + a_3} \geq 0$.

Mihály Bencze

**OQ. 3649.** Determine all $\alpha \geq \beta > 0$ and $n \in N$ $(n \geq 3)$ such that
$\sum\limits_{k=1}^{n} \frac{a_k^\alpha}{a_1^\alpha + ... + a_{k-1}^\alpha + a_{k+1}^\alpha + ... + a_n^\alpha} \geq \sum\limits_{k=1}^{n} \frac{a_k^\beta}{a_1^\beta + ... + a_{k-1}^\beta + a_{k+1}^\beta + ... + a_n^\beta}$ for all $a_k > 0$ $(k = 1, 2, ..., n)$.

Mihály Bencze

**OQ. 3650.** 1). Determine all $n \in N$ such that
$\left(\frac{\sum\limits_{k=1}^{n} a_k^3}{\sum\limits_{k=1}^{n} a_k^2}\right)^{12} \geq \left(\frac{\sum\limits_{k=1}^{n} a_k^3}{\sum\limits_{k=1}^{n} a_k}\right)^{6} \geq \left(\frac{1}{n}\sum\limits_{k=1}^{n} a_k^4\right)^3$ for all $a_k > 0$ $(k = 1, 2, ..., n)$. For $n = 2$ it is true.
2). Determine all $a_k > 0$ $(k = 1, 2, ..., n)$ such that
$\left(\frac{\sum\limits_{k=1}^{n} a_k^3}{\sum\limits_{k=1}^{n} a_k^2}\right)^{12} \geq \left(\frac{\sum\limits_{k=1}^{n} a_k^3}{\sum\limits_{k=1}^{n} a_k}\right)^{6} \geq \left(\frac{1}{n}\sum\limits_{k=1}^{n} a_k^4\right)^3$ for all $n \in N^*$

Mihály Bencze

**OQ. 3651.** Determine all $x_k > 0$ $(k = 1, 2, ..., n)$ such that
$\sum\limits_{k=1}^{n} \frac{1}{(1+x_k)^n} \geq \frac{1}{1+\prod\limits_{k=1}^{n} x_k}$.

Mihály Bencze

**OQ. 3652.** Determine all $\alpha > 0$ such that $\sum \left(\frac{a(b+c)}{a^2+bc}\right)^\alpha \geq 2$, for all $a, b, c > 0$. If $\alpha \in \left\{1, \frac{1}{2}\right\}$ the affirmation is true.

Mihály Bencze

**OQ. 3653.** 1). If $a_k > 0$ $(k = 1, 2, ..., n)$ and $A_n = \frac{1}{n}\sum\limits_{k=1}^{n} a_k$, $G_n = \sqrt[n]{\prod\limits_{k=1}^{n} a_k}$, $H_n = \frac{n}{\sum\limits_{k=1}^{n} \frac{1}{a_k}}$, then $\frac{n}{H_n} + \frac{4}{A_n+G_n} \geq \frac{n+2}{G_n}$
2). Determine all $x, y > 0$ such that $\frac{n}{H_n} + \frac{2(x+y)}{xA_n+yG_n} \geq \frac{n+2}{G_n}$
3). Determine all means $M_1(a_1, a_2, ..., a_n)$, $M_2(a_1, a_2, ..., a_n)$, $M_3(a_1, a_2, ..., a_n)$ for which $M_1 \geq M_2 \geq M_3$ and $\frac{n}{M_3} + \frac{4}{M_1+M_2} \geq \frac{n+2}{M_2}$
4). Determine all $x, y > 0$ such that $\frac{n}{M_3} + \frac{2(x+y)}{xM_1+yM_2} \geq \frac{n+2}{M_2}$.

Mihály Bencze

**OQ. 3654.** 1). If $a_k > 0$ $(k = 1, 2, ..., n)$ and $A_n = \frac{1}{n}\sum\limits_{k=1}^{n} a_k$, $G_n = \sqrt[n]{\prod\limits_{k=1}^{n} a_k}$,
$H_n = \frac{n}{\sum\limits_{k=1}^{n}\frac{1}{a_k}}$, then $\left(\frac{nA_n-(n-1)G_n}{G_n}\right)^{n-1} \geq \frac{nG_n-(n-1)H_n}{H_n}$
2). Determine all $x > 0$ such that $\left(\frac{(x+1)A_n-xG_n}{G_n}\right)^{x} \geq \frac{(x+1)G_n-xH_n}{H_n}$
3). Determine all means $M_1(a_1, a_2, ..., a_n)$, $M_2(a_1, a_2, ..., a_n)$, $M_3(a_1, a_2, ..., a_n)$ such that $M_1 \geq M_2 \geq M_3$ and
$\left(\frac{nM_1-(n-1)M_2}{M_2}\right)^{n-1} \geq \frac{nM_2-(n-1)M_3}{M_3}$.
4). Determine all $x > 0$ such that $\left(\frac{(x+1)M_1-xM_2}{M_2}\right)^{x} \geq \frac{(x+1)M_2-xM_3}{M_3}$.

Mihály Bencze

**OQ. 3655.** If $a, b, c > 0$ then determine all $x \geq 0$ such that
$\sum\frac{1}{a^2+xbc} \geq \frac{x}{\sum ab} + \frac{1}{\sum a^2}$.
If $x = 2$ then it is true.

Mihály Bencze

**OQ. 3656.** Let $ABC$ be a triangle with sides $a, b, c$. Determine all $\alpha, \beta > 0$ such that $\sum\limits_{cyclic}\frac{a^{2\alpha}-(bc)^{\alpha}}{3a^{2\alpha}+b^{2\alpha}+2^{2\alpha}} \leq 0 \leq \sum\limits_{cyclic}\frac{a^{2\beta}-(bc)^{\beta}}{3a^{2\beta}+b^{2\beta}+2^{2\beta}}$. If $\alpha = 1$ and $\beta = 2$, then we have a solution.

Mihály Bencze

**OQ. 3657.** If $a, b, c > 0$ then determine all the functions $f : R \to (0, +\infty)$ such that $\frac{1}{xa^2+bc} + \frac{1}{xb^2+ca} + \frac{1}{xc^2+ab} \geq \frac{f(x)}{(a+b+c)^2}$. We have find that $f(1) = 12$ and $f(2) = 8$.

Mihály Bencze

**OQ. 3658.** 1). Let be $x_1 = x_2 = 1$ and $x_{n+2} = ax_{n+1} + bx_n$ for all $n \geq 1$. Determine all $a, b \in Z$ for which $(x_n)_{n\geq 1}$ contains an infinite increasing sequence such that every two terms of this sequence are relatively prime.
2). Let be $x_1 = x_2 = ... = x_k = 1$ ans $x_{n+k} = a_1x_n + a_2x_{n+1} + ... + a_kx_{n+k-1}$ for all $n \geq 1$. Determine all $a_1, a_2, ..., a_k \in Z$ for which $(x_n)_{n\geq 1}$ contains an infinite increasing sequence such that every $k$ terms of this sequence are relatively primes.

Mihály Bencze

**OQ. 3659.** 1). Determine all $n \in N^*$ for which $(n, 2^n - 1)$ is prime.
2). Exist infinitely many $n \in N$ for which $(n, 2^n - 1)$ is prime?

Mihály Bencze

**OQ. 3660.** Determine all prime $p$ for which $\left(n, 2^{2^n} + p\right) = p$ for all $n \in N^*$.

Mihály Bencze

**OQ. 3661.** Solve in $N$ the equation $x(x+1)...(x+k) = y^{k+1} + 1$.

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**OQ. 3662.** Find all $x_k \in Q$ $(k = 1, 2, ..., n)$ such that $\frac{x_1}{x_2} + \frac{x_2}{x_3} + ... + \frac{x_n}{x_1} = 1$.

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**OQ. 3663.** Determine all $y, x_k \in N$ $(k = 1, 2, ..., n)$ and $m \in N$ such that
$1 + \sum\limits_{k=1}^{n} x_k^m = y^m$.

Mihály Bencze

**OQ. 3664.** Determine all $x_k, y_k \in N$ $(k = 1, 2, 3)$ such that
$\begin{cases} x_1 + x_2 + x_3 = y_1 y_2 y_3 \\ y_1 + y_2 + y_3 = x_1 x_2 x_3 \end{cases}$.

Mihály Bencze

**OQ. 3665.** Determine all $x, y, n \in N$ such that $(x+1)^n = x^n + y^{n-1}$.

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